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Pseudo-Generic Products and Mergers in Pharmaceutical Markets

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Abstract

This paper fills the gap in the theoretical literature concerning mergers between brand-name and generic laboratories in pharmaceutical markets. To prevent generic firms from increasing their market share, some brand-name firms produce generics themselves, called pseudo-generics, enabling them to set up barriers to entry. We develop this topic by considering the pseudo-generics production as a mergers' catalyst. We show, in a duopoly model with substitutable goods, in which a brand-name firm and a generic firm compete *à la* Cournot, that a brand-name company always has an incentive to purchase its competitor. The key insight of this paper is that the brand-name laboratory can increase its merger gain by producing pseudo-generics beforehand. In some cases, pseudo-generics would not otherwise be produced.

JEL classification: I11, L12.

Keywords: Mergers, Pharmaceutical Market, Pseudo-Generics.

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1 Introduction

In the pharmaceutical market, drug producers apply for patents in order to protect their intellectual property rights. At the patents expiration date, these rights become public property. The production of generic goods, manufactured with the same molecules as the brand-name drugs, can then start to compete with the original product. Generic drugs are certified to be "therapeutically equivalent" to the originator's product. But, for consumers, they may still vary in characteristics such as shape, color, flavor, scoring, packaging, labelling, shelf life and brand loyalty[1]. Therefore, these goods are not considered as perfect substitutes. The generics market development is an answer from insurers[2] to the increase in drug spending. In 2006, the expected global generic growth rate is about 7 % (IMS-Health). Caves et alii (1991), Grabowski and Vernon (1992), Frank and Salkever (1992, 1997), Morton (1999, 2000) analyze the effect of generics entry on prices and market shares of brand-name products in the United States[3]. However, they do not consider pseudo-generics.The pseudo-generic is identical to the brand-name product, but is marketed as a generic. In Australia and Canada, pseudo-generics have about one quarter of the generic market. They are also in a strong position in New Zealand, Germany, the UK and Sweden (Hollis (2002)). The significant pseudo-generics production explains our interest in this topic. Indeed, this paper analyzes the pseudo-generics production effects on merger strategies. Thus, the paper fill the gap in the theoretical literature on mergers between brand-name and generic laboratories. In particular, we consider pseudo-generics production as a mergers' catalyst.

The economists' interest in pseudo-generics effect on the pharmaceutical market is recent, both at an empirical and theoretical level. At the empirical level, Hollis (2002) is the first paper to consider the effect of pseudo-generics on prices and generics entry. He shows the presence of a first mover advantage. This advantage deters entry and leads to the increase in prices, both for pseudo-generics and brand-name products. Hollis (2002) concludes that the welfare decreases on the Canadian market. At the theoretical level, Ferrandiz (1999) analyzes the decision to produce pseudo-generics in a market where branded goods are perfect substitutes and where there exists a degree of product differentiation between brand-name drugs and generics. He shows in a model, via simulation outcomes, that it is better for a brand-name firm to produce pseudo-generics than to accept the entry of the generic firm. The brand-name firm takes this decision in order to increase the brand-name price[4] and its global profit, owing to a market segmentation effect. On the other hand, Kong and Seldon (2004) use a two-stage game model with product differentiation. They find that, if the cross-price effect between the brand-name product and its generic equivalent is sufficiently large, the brand-name incumbent produces the pseudo-generic drug, increasing its profit and deterring generic entry. From a policy perspective, the results of this research imply that: either the pseudo-generic should be banned, or

[1]Due to the trademark protection, the generic manufacturers may not be allowed to produce generic versions that have exactly the same apparence as the brand-name originals (Ching (2000)).

[2]The development of the generics market is favored by the implementation of a right of substitution, of an international nonproprietary name, of formularies, of controls on prescribing doctors and pharmacists, and of controls on drug prices, notably.

[3]Some companies see more than 20 % of their sales threatened by the generic entry on their market (Grandfils and alii (2004)).

[4]Frank and Salkever (1992) is the first paper modelizing the price increase of the branded good when the generic drug enters in the pharmaceutical market.

a period without pseudo-generic should be guaranteed to the first generic firm[5] (Hollis (2003)).

There have been few theoretical studies on mergers between brand-name and generic laboratories. This paper is an attempt to fill that gap. In particular, we consider pseudo-generics production as a merger catalyst. In other words, we study how merger stategies can influence pseudo-generics production. We modelize an industry in which a brand-name firm and a generic firm compete *à la* Cournot. Brand-name and generic goods are considered as imperfect substitutes. This model presents a two-stage non-cooperative game. At each stage, the brand-name firm can purchase its generic competitor. But, in the first stage, the brand-name firm can decide to produce pseudo-generics instead. In this framework, the generic laboratory is a Stackelberg leader on the generic market where two homogeneous goods are available: generic and pseudo-generic products. The pseudo-generics production at the initial stage reduces the cost of the merger at the second stage. Indeed, the competition increases and lowers the purchase price of the generic firm. We find three results. First, without pseudo-generics, the brand-name producer always monopolizes the market. Second, the pseudo-generics production may delay the takeover. Indeed, under some conditions, the "delayed" merger dominates the first period merger. Third, the brand-name company can produce pseudo-generics solely to monopolize the industry, even if it is not profitable at first. Indeed, under some market conditions, the pseudo-generics production reduces the brand-name firm's profit, but this firm nonetheless decides to produce this pseudo-generic in order to lower the purchase price of the generic manufacturer. Two conditions must be satisfied to insure the profitability of this strategy: the loss incurred by the pseudo-generics production over a period must not be too significant. Moreover, the discounted value of the gain associated with the reduction in the purchase price must be high enough.

In our paper, we extend Kong and Seldon (2004). The latter is the only paper concerning both anti-competitive practices in the pharmaceutical market and pseudo-generics production. Our treatment differs on two points. On one hand, we consider that the generic product enters before the pseudo-generic product, and so, benefits from the generic market leadership. This market structure better reflects the future evolution of the generic market (Hollis (2003)). On the other hand, the monopolization of the pharmaceutical market can thus take place only by mergers because the generic product is present in the market before the pseudo-generic product. So, we study mergers and not barriers to entry, contrary to Kong and Seldon (2004).

The rest of the paper is organized into five sections. Section 2 sets up the basic model. Section 3 shows how merger incentives are modified with the pseudo-generic entry. In section 4, we analyze effects of merger strategies on pseudo-generics production. Finally, section 5 concludes.

2 Basic model

Firstly, we present the assumptions of the model. Then, we study the benchmark. We start by investigating the conditions under which the duopolistic situation is sustainable,

[5] In the United States, the 1984 Hatch-Waxman Act, guaranteed a period of six months of exclusivity from the date it starts marketing its generic drug. This patent is set up to favour the generic entry (Eco-Santé OCDE (2001)).

assuming that pseudo-generics can not be produced. In this framework, we complete the study by analyzing the incentives to merge.

2.1 Assumptions

We study a two-firm industry in which a brand-name laboratory and a generic firm compete *à la* Cournot. By assumption, the price-elasticity in the generic market is higher than the price-elasticity in the branded market as in Frank et Salkever (1992). The utility results from the satisfaction removed from the consumption of the quantity q_b of brand-name goods and the quantity Q_G of its generic substitute, with $Q_G = q_g + q_{pg}$. Note that q_g is the generic quantity and q_{pg} is the pseudo-generic quantity. We assume that, for consumers, generic and pseudo-generic goods are homogeneous (Hollis (2002)). In a compromise between generality and tractability, we assume that the quadratic utility function of the representative consumer is the following one:

$$\begin{aligned} U &= V_0 + U(q_b, Q_G) \\ &= V_0 + \zeta_b q_b + \zeta_G Q_G - \frac{1}{2}(\,\alpha\, q_b^2 + 2\gamma\, q_b\, Q_G + \beta\, Q_G^2), \end{aligned} \tag{1}$$

where V_0 reflects the utility derived from a competitive numeraire sector. We assume $\zeta_b > 0$, $\zeta_G > 0$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$, and $\alpha < \beta$ to take into account the preference for brand-name products. To simplify and without loss of generality, we normalize the size of the market to unity, *i.e* $\zeta_G = \zeta_b = 1$. To insure the concavity of utility and profit functions, we assume $\alpha\beta > \gamma^2$. $\frac{\gamma^2}{\alpha\beta}$ expresses the degree of product differentiation (as $\alpha < \beta$, we often compare α and γ to analyze product differentiation).

From (1), one can derive linear inverse demand relations:

$$p_G = \frac{\partial U}{\partial Q_G} = 1 - \beta\, Q_G - \gamma\, q_b. \tag{2}$$

$$p_b = \frac{\partial U}{\partial q_b} = 1 - \alpha\, q_b - \gamma\, Q_G. \tag{3}$$

Finally, we assume that there is no production cost. This assumption makes it easier to capture the product differentiation effects on firms' strategies.

2.2 Duopoly without pseudo-generic products

We study the conditions under which the duopolistic situation is sustainable, assuming that pseudo-generics can not be produced. We assume that a brand-name laboratory and a generic firm compete *à la* Cournot. Let Π_b^D be the brand-name firm's profit function and Π_g^D be the generic laboratory's profit function. Note that $Q_G = q_g$ in this case.

We derive the equilibrium outputs from the first order conditions: (see appendix A)

$$q_g^{D*} = \frac{2\alpha - \gamma}{4\alpha\beta - \gamma^2}. \tag{4}$$

$$q_b^{D*} = \frac{2\beta - \gamma}{4\alpha\beta - \gamma^2}. \tag{5}$$

Remark 1 *The duopoly exists if product differentiation is high enough ($\gamma < 2\alpha$).*

On the brand-name market, if product differentiation is not too high ($\gamma > 2\alpha$), the own-price effect on the brand-name product is very low compared to the own-price effect on the generic product and to the cross-price effect. Therefore, brand-name goods are produced in very large quantities. Thus, the price is negative in the generic market and the generic firm has no incentive to produce.

We derive the equilibrium profits:

$$\Pi_b^{D*} = \frac{\alpha(2\beta - \gamma)^2}{(4\alpha\beta - \gamma^2)^2}. \tag{6}$$

$$\Pi_g^{D*} = \frac{\beta(2\alpha - \gamma)^2}{(4\alpha\beta - \gamma^2)^2}. \tag{7}$$

2.3 Merger without pseudo-generic products

Here, we are interested in the possibility that the brand firm acquires the generic firm in order to monopolize the market[6]. Thus, the following non-cooperative game is played: the brand-name firm chooses between two events. Either merge with the generic firm, or stay in the duopolistic industry. We call status quo this second event (see figure 1 for the extended form of the game).

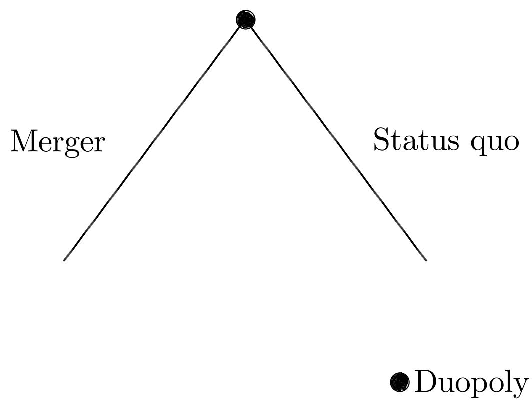

Figure 1

In this game, the brand-name firm makes a bid for the generic firm. The generic firm gives a reservation price below which it refuses to be sold. If the bid is superior or equal to the reservation price, the brand-name laboratory purchases the generic firm. Otherwise, the status quo is equilibrium of this game.

We look for the Nash equilibrium of this game. Therefore, we determine the incentives to merge. Firstly, we study the monopoly profit. Indeed, the merger incentive depends on the difference between the monopoly profit and the merger costs.

[6]Two reasons rule out the possibility that the generic firm initiate the takeover. First, no generic firm bought back a brand-name firm to this day. Second, it is easier for the brand-name firm to initiate the takeover. Indeed, it benefited from a monopolistic situation thanks to its patent and because of the brand-name demand inelasticity.

2.3.1 The monopoly, goal of the merger

In order to determine the equilibrium of the game, we need the monopoly profit function. Indeed, the merger leads to this market structure. In the previous section, we showed that a *de facto* monopoly exists if and only if the product differentiation is not too high ($\gamma > 2\alpha$). For a brand-name market price-sensitivity below this threshold ($\gamma > 2\alpha$), the generic firm exits the market. The monopoly can also exist because of a merger. This is possible if product differentiation is high enough ($\gamma < 2\alpha$).

Let us study the case where the branded firm has a monopoly power and produces generics (pseudo-generics). Let us note its profit Π_b^M.

We derive the equilibrium outputs from the first order conditions(see appendix B):

$$q_b^{M*} = \frac{1}{2}\frac{\beta - \gamma}{\alpha\beta - \gamma^2}. \tag{8}$$

$$q_g^{M*} = \frac{1}{2}\frac{\alpha - \gamma}{\alpha\beta - \gamma^2}. \tag{9}$$

We derive the equilibrium profits:

$$\Pi_b^{M*} = \frac{1}{4}\frac{\beta - 2\gamma + \alpha}{\alpha\beta - \gamma^2}. \tag{10}$$

If product differentiation is not too high ($\alpha < \gamma$), the monopoly produces exclusively brand-name goods. Let Π_b^{MM} be its profit function. We derive the equilibrium output from the first order condition (see appendix C):

$$q_b^{MM*} = \frac{1}{2\alpha}. \tag{11}$$

We derive the equilibrium profit :

$$\Pi_b^{MM*} = \frac{1}{4\alpha}. \tag{12}$$

Remark 2 *If product differentiation is high enough ($\gamma < \alpha$), the monopoly produces brand-name and generic goods. Indeed, inter-market competition is not too intensive.*

Intuitively, if product differentiation is not too high ($\gamma > \alpha$), the own-price effect in the brand-name market is lower than the own-price effect in the generic market and the cross-price effect. Thus, there is no incentive to produce generics. Indeed, to produce generics would decrease the monopoly profit earned on the brand-name product. Moreover, the profit earned on the generic product would not compensate this loss.

2.3.2 Merger incentive

In order to determine brand-name firm's merger incentive, we compare its merger payoff and its status quo payoff. Let $\Sigma_1(\alpha, \beta, \gamma)$ be the merger gain and $\sigma_1(\alpha, \beta, \gamma)$ the opportunity cost lost by the firm when it merges, *i.e* the status quo payoff.

The merger gain is the difference between the monopoly profit and the buying price. According to the range of demand parameters, there are two monopoly profit functions (see section 2.3.1). The takeover takes place if and only if the brand-name firm's bid is superior or equal to the reservation price. The generic firm always sets the same reservation price, equal to its duopoly profit[7]. Indeed, we assume that it can not ask more because, refusing to sell, it realizes this duopoly profit. In order to simplifly the analysis, we assume that a bid equal to the reservation price makes the generic firm indifferent between whether to proceed with the sale or not. In this case, it chooses to be sold. Thus, in order to merge, the brand-name firm makes a bid equal to the reservation price, i.e generic firm's duopoly profit. In the merger case, the purchase price is then equal to the duopoly profit. The merger gain is:

$$\Sigma_1(\alpha,\beta,\gamma) = \begin{cases} \Pi_b^{MM*} - \Pi_g^{D*} \text{ in the case where } \frac{\gamma}{2} < \alpha < \gamma. \\ \Pi_b^{M*} - \Pi_g^{D*} \text{ in the case where } \alpha > \gamma. \end{cases} \tag{13}$$

Concerning the opportunity cost, it is equal to profit given up by the brand-name firm when the latter decides to merge, i.e its duopoly profit (status quo):

$$\sigma_1(\alpha,\beta,\gamma) = \Pi_b^{D*}. \tag{14}$$

Therefore, there is an incentive to merge if and only if:

$$F(\alpha,\beta,\gamma) = \Sigma_1(\alpha,\beta,\gamma) - \sigma_1(\alpha,\beta,\gamma) \geq 0. \tag{15}$$

We show that $F(\alpha,\beta,\gamma) \geqslant 0$ for $\alpha \in \left[\frac{\gamma}{2},\beta\right[$.

Proof. see appendix D. ■

Remark 3 *In the duopoly case, the merger always takes place. We claim that the equilibrium of this game is the merger situation.*

The monoplization leads to an increase in market power and then to an increase in profits. This compensates for generic firm's buying price. Indeed, if the goods were homogeneous, the market power gain would be higher than the loss related to the buying price (Kamien and Zang (1990)). In our study, goods are not perfect substitutes. Thus, there is a drop in the merger gain since the generic market is more sensitive to price. But, on the other hand, generic firm's buying price also decreases since generic firm's profit is lower than if this firm produces the homogeneous product. This shows that results of Kamien and Zang (1990) are robust if one takes away the perfectly substitutable goods assumption.

Now, we show that pseudo-generics production possibility may modify incentives to merge.

3 Pseudo-generic and "delayed" merger

In this section, we consider that the brand-name laboratory can produce a pseudo-generic good. We study incentives to merge within this framework. If a takeover takes place after pseudo-generics production, it is called "delayed" merger. The brand-name firm maximizes the sum of its profit earned on brand-name goods and its profit earned on pseudo-generic

[7] The unique strategy of the generic firm is to give an asking price equal to Π_g^{D*}.

goods. In this case, we recall that $Q_G = q_g + q_{pg}$ where q_g is the generic output and q_{pg} the pseudo-generic output and that, for consumers, generic and pseudo-generic goods are homogeneous (Hollis (2002)). In the rest of the paper, we call "pseudo-duopoly" the industry structure in which the brand-name firm produces pseudo-generics.

For the benchmark case, we showed that the brand-name firm monopolizes the industry. Another strategy is now possible: the brand-name firm can produce pseudo-generic goods. Consequently, we consider a second step: having produced pseudo-generics during the first production period, the brand-name firm can either continue to produce pseudo-generics, or merge with the generic firm. Therefore, we consider a two-stage game:

In the first stage, there are three states of the world: the two previous states (*i.e* the merger and the status quo) and a new one in which the brand-name firm decides to produce pseudo-generics. After this stage, a production period takes place. In the second stage, if the brand-name produced pseudo-generics, two states of the world are available: merger or pseudo-duopoly[8]. Otherwise, the industry structure does not change. After this stage, competition takes place on an infinite horizon.

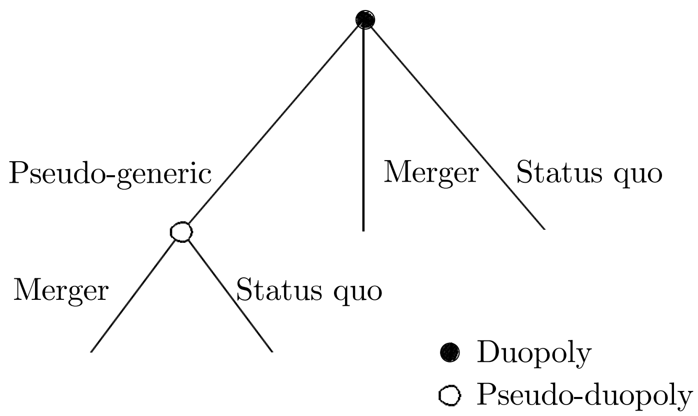

Figure 2

We solve backward the non-cooperative game in order to compute sub-game perfect Nash equilibria (SPNE). In the second stage of the game, we determine the sub-game equilibrium: we find the equilibrum between to merge or not, pseudo-generics production being decided at the first stage (section 3.2). Finally, we find the SPNE of the game (section 3.3). In order to solve the game, we previously determine the pseudo-duopoly profit functions (section 3.1).

3.1 Pseudo-generics production

We have to study the market structure in which the brand-name firm produces pseudo-generics: the pseudo-duopoly structure. Indeed, the two firms compete *à la* Cournot but the brand-name firm produces two drugs: brand-name and pseudo-generic goods. Obviously, the generic firm produces the generic good. Furthermore, the generic is present in the market before the pseudo-generic firm's entry, thus benefiting from the first mover

[8] After pseudo-generics production, the merger can only take place at the second stage of the game (see section 3.3 for more details)

advantage. This advantage confers on it the generic market leadership. So, we talk about Cournot-Stackelberg competion in this pseudo-duopolistic industry. Let Π_g^P be generic laboratory's profit and Π_b^P be brand-name laboratory's profit.

To compute the equilibrium profits, it is necessary to determine the reaction function $RF(q_b, q_g)$ concerning pseudo-generics (see appendix E). Substituting $RF(q_b, q_g)$ in both profit functions, we derive the equilibrium outputs from the first order conditions:

$$q_g^{P*} = \frac{1}{2\beta}. \tag{16}$$

$$q_b^{P*} = \frac{1}{2}\frac{\beta-\gamma}{\alpha\beta-\gamma^2}. \tag{17}$$

Substituting (16) and (17) in $RF(q_b, q_g)$, we derive the pseudo-generics equilibrium output:

$$q_{pg}^{P*} = \frac{1}{4}\frac{\alpha\beta - 2\gamma\beta + \gamma^2}{\beta(\alpha\beta-\gamma^2)}. \tag{18}$$

We deduce the equilibrium profits:

$$\Pi_g^{P*} = \frac{1}{8\beta}. \tag{19}$$

$$\Pi_b^{P*} = \frac{1}{16}\frac{\alpha\beta + 4\beta^2 - 8\gamma\beta + 3\gamma^2}{\beta\left(\alpha\beta-\gamma^2\right)}. \tag{20}$$

The pseudo-duopoly exists if the pseudo-generics equilibrium output is positive. This is the case if:

$$\alpha > A(\beta,\gamma), \text{ with } A(\beta,\gamma) = \frac{\gamma(2\beta-\gamma)}{\beta}. \tag{21}$$

To solve for SPNE of the game, we compute the sub-game equilibrium of the second stage. In particular, we determine the merger incentive in the pseudo-duopolistic structure.

3.2 Second stage: merger incentive in the pseudo-duopoly market

We study incentives to merge in the case where the industry is initially pseudo-duopolistic. These incentives change compared to the duopoly case. In this framework, the brand-name producer chooses between two events. It can merge or stay in the same market structure. In the first case, it monopolizes the market and produces brand-name goods and generics because the product differentiation is high enough ($\alpha > \gamma$: see remark 2). Indeed, the pseudo-duopoly structure can exist only if $\alpha > A(\beta,\gamma)$. Moreover, $A(\beta,\gamma) > \gamma$. The generic firm chooses between two strategies: to accept the buying price or to reject it. This firm always sets a reservation price equal to its profit in the status quo case, *i.e* its pseudo-duopoly profit (Π_g^{P*}). In order to merge, we assume that the brand-name firm has only to make a bid equal to the reservation price of the generic firm.

The brand-name firm has an incentive to acquire the generic producer if its merger gain is higher than its opportunity cost. Let $\Sigma_2(\alpha,\beta,\gamma)$ be the merger gain:

$$\Sigma_2(\alpha,\beta,\gamma,\delta) = \Pi_b^{M*} - \Pi_g^{P*}. \tag{22}$$

Let $\sigma_2(\alpha, \beta, \gamma, \delta)$ be the opportunity cost. In other words, it is about brand-name firm's profit when this firm does not merge (status quo):

$$\sigma_2(\alpha, \beta, \gamma, \delta) = \Pi_b^{P*}. \tag{23}$$

Thus, the merger incentive exists if and only if:

$$\Sigma_2(\alpha, \beta, \gamma, \delta) - \sigma_2(\alpha, \beta, \gamma, \delta) = \frac{\Pi_b^{M*} - \Pi_g^{P*} - \Pi_b^{P*}}{1-\delta} \geq 0. \tag{24}$$

The condition (27) is always verified[9], thus :

Remark 4 *In the pseudo-duopoly regime, the merger always takes place.*

We claim that the Nash equilibrium of this sub-game is the merger situation. The merger is possible whatever the demand parameters are. The buying price of the generic firm is lower in the pseudo-duopoly regime than in the duopoly regime because the generic firm is directly competed against. Indeed, generic and pseudo-generic goods are perfect substitutes.

Given the sub-game equilibrium found in this section, we turn to the first stage of the game to compute the SPNE.

3.3 First stage: the "Delayed" merger incentive

Relative to section 2, a new path is analyzed. In section 3.2, we solved the sub-game of the second stage. Whatever demand parameters are selected, there is an unique Nash equilibrium: the monopolization of the pseudo-duopoly industry. Therefore, the new path to study is the "delayed" merger (*i.e* to merge after having produced pseudo-generics). For the brand-name laboratory, the sole interest in this path is to reduce the purchase price of the generic firm. Nevertheless, the brand-name firm gives up the monopoly profit during a production period. Indeed, it would earn the monopoly profit at once by merging before the first production period[10].

We determined the equilibrium of the second stage in the case where the pseudo-generic drug has been produced at the first stage. Without pseudo-generics, the merger event dominates the status quo event. Thus, there is always monopolization (see section 2.3). In order to find the SPNE of the two-stage game, we must compare two branches of the game tree: either the merger at the beginning of the game, or the "delayed" merger.

The repurchase mechanism is the same as previously. Here, the takeover takes place at the beginning of the second period. We study this new path. The brand-name firm makes a bid for the generic firm. The latter gives a reservation price below which it rejects the offer. If the bid is superior or equal to the reservation price, the takeover takes place. As there is a two-stage game and as the horizon is infinite, payoffs must take into account the rate of discount δ[11].

[9] Indeed, $\Pi_b^{M*} - \Pi_g^{P*} - \Pi_b^{P*} = \frac{1}{16\beta} > 0$.

[10] Without merger, we exclude pseudo-generics production for more than one period. Indeed, the generic firm's buying price is constant once the pseudo-generic good is produced. Moreover, by merging later, the brand-name firm gives up the monopoly profit for a longer time.

[11] $\delta = \frac{1}{1+r}$, with r the interest rate.

In this section, we determine the "delayed" merger gain. This is the payoff earned by the brand name firm, following this merger path. This path can be analyzed in two steps because there is a change between the first production period and the following ones. During the first period, the brand-name firm earns a pseudo-duopoly profit. After, it gets a monopoly profit[12] for an infinite horizon. In order to merge, the brand-name firm has to pay the generic firm's pseudo-duopoly profit for the following periods. Let $\Sigma_3(\alpha, \beta, \gamma, \delta)$ be the "delayed" merger gain:

$$\Sigma_3(\alpha, \beta, \gamma, \delta) = \Pi_b^{P*} + \frac{\delta \Pi_b^{M*}}{1-\delta} - \frac{\delta \Pi_g^{P*}}{1-\delta}. \tag{25}$$

In the rest of the paper, the first stage merger is called "standard" merger. In section 2.3, we showed that the "standard" merger is preferred to the status quo. Thus, we have to compare the "delayed" merger gain and the "standard" merger gain. Note that, in the previous section, we computed the "standard" merger gain in a static framework and for a high enough level of product differentiation ($\alpha > \frac{\gamma}{2}$, see equality (13)). Now, we restrict the range of parameters because pseudo-generics production may exist only if $\alpha > A(\beta, \gamma)$. Moreover, as the horizon is infinite, the "standard" merger gain is the discounted infinite flow of monopoly profit minus the buying price (*i.e* the infinite flow of generic firm's duopoly profit). Therefore, we compare the "delayed" merger gain and the discounted infinite flow of "standard" merger gain ($\frac{\Sigma_1(\alpha,\beta,\gamma)}{1-\delta}$), with $\alpha > A(\beta, \gamma)$. Let $S(\alpha, \beta, \gamma, \delta)$ be the difference between these two gains:

$$S(\alpha, \beta, \gamma, \delta) = \Sigma_3(\alpha, \beta, \gamma, \delta) - \frac{\Sigma_1(\alpha, \beta, \gamma)}{1-\delta} = \Pi_b^{P*} - \Pi_b^{M*} + \frac{\Pi_g^{D*}}{1-\delta} - \frac{\delta \Pi_g^{P*}}{1-\delta}. \tag{26}$$

If $S(\alpha, \beta, \gamma, \delta) > 0$, then the "delayed" merger is SPNE of the game. Let us detail $S(\alpha, \beta, \gamma, \delta)$. Note $c(\beta) = \Pi_b^{M*} - \Pi_b^{P*} = \frac{3}{16\beta}$. This is the cost due to the "delayed" merger compared to the "standard" merger. This cost is borne during the first period. Note $g(\alpha, \beta, \gamma, \delta) = \frac{\Pi_g^{D*}}{1-\delta} - \frac{\delta \Pi_g^{P*}}{1-\delta}$. This is the decrease in the purchase price associated with the "delayed" merger event. Indeed, instead of paying the discounted infinite flow of the generic firm's duopoly profit from the beginning, the brand-name firm pays, at the second stage, the discounted infinite flow of the generic firm's pseudo-duopoly profit.

Proposition 1 *The "delayed" merger is the SPNE of this game if and only if:*
(i) the own-price effect for the brand-name product is relatively high,
(ii) the own-price effect for the generic product is rather strong compared to the cross-price effect,
(iii) the rate of discount is sufficiently high (i.e $\delta > \overline{\overline{\delta}}(\alpha, \beta, \gamma)$).

Proof. We show that $S(\alpha, \beta, \gamma, \delta) > 0$ for :
-$\alpha > \alpha_2(\beta, \gamma)$,
-$\beta > \gamma(\frac{1}{2} + \frac{\sqrt{2}}{2})$,

[12] The pseudo-generic good may be produced only if $\alpha > A(\beta, \gamma)$. Thus, $A(\beta, \gamma) > \gamma$. We deduce that $\alpha > \gamma$ in this framework. Therefore, the monopoly produces two substituable goods.

-$\delta \in \left]\overline{\overline{\delta}}(\alpha,\beta,\gamma),1\right[$, with $\overline{\overline{\delta}}(\alpha,\beta,\gamma) = \frac{\left(-16\beta^2\right)\alpha^2+\left(64\beta^2\gamma-24\beta\gamma^2\right)\alpha+\left(3\gamma^4-16\beta^2\gamma^2\right)}{(4\alpha\beta-\gamma^2)^2}$.

(see appendix F for the study of $c(\beta)$ and the study of $g(\alpha,\beta,\gamma,\delta)$. See appendix G for the study of $S(\alpha,\beta,\gamma,\delta)$). ■

Intuitively, if the own-price effect for the brand-name product is relatively high, then the "delayed" merger gain increases compared to the "standard" merger gain. Indeed, the generic duopoly profit increases with the brand-name market price-sensitivity (α) while the pseudo-duopoly profit is independent of this sensitivity. Furthermore, if the generic market price-sensitivity (β) is high enough, the first period loss due to the "delayed" merger is relatively low. Under these parameter conditions, there is a rate of discount high enough such as the earnings of the second period (the "delayed" merger purchase price is lower than the "standard" merger one: $\Pi_g^{P*} < \Pi_g^{D*}$) exceeds the loss of the first period.

Now, we study how the profitability zone of "delayed" mergers ($\delta \in \left]\overline{\overline{\delta}}(\alpha,\beta,\gamma),1\right[$) is affected by demand shocks.

We show that $\frac{\partial\overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial\alpha} < 0$, $\frac{\partial\overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial\beta} > 0$, and $\frac{\partial\overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial\gamma} > 0$.

Proof. see appendix H. ■

Remark 5 *The "strategic" merger profitability increases (respectively decreases) when the brand-name market price-sensitivity (α) increases, given the generic market price-sensitivity (β) and the cross price-sensitivity (γ) (respectively when β increases given α and γ or when γ increases given α and β).*

Intuitively, the minimum rate of discount permitted to achieve a "delayed" merger decreases according to the price-sensitivity increase in the brand-name market, given other demand parameters. In other words, this relative increase in the price-sensitivity in the brand-name market allows the gain in purchase price due to the "delayed" merger to increase. So, the development of this merger gain does not need to be very high to compensate for the loss undergone during the first period since the brand-name firm does not merge at the first stage. This means that the "delayed" merger is profitable for low rates of discount when the price-sensitivity in the brand-name market is high.

We show that $\overline{\overline{\delta}}(\overline{\overline{\alpha}}(\beta,\gamma),\beta,\gamma) = 0$.

Proof. see the discounted rate neutrality in appendix I. ■

Remark 6 *There exists a threshold $\overline{\overline{\alpha}}(\beta,\gamma)$ (a very high product differentiation) such as if $\alpha > \overline{\overline{\alpha}}(\beta,\gamma)$, then $\overline{\overline{\delta}}(\alpha,\beta,\gamma) \leq 0$. In this extreme case, the "delayed" merger takes place whatever the rate of discount is.*

4 "Delayed" merger and "Strategic" merger

In this section, we focus on merger strategy effects on pseudo-generics production. Therefore, we analyze the pseudo-generics production decision when mergers are not considered (section 4.1). Afterwards, relating to mergers, we show there is an incentive to produce pseudo-generics that did not previously exist. This merger is called "strategic" merger. Indeed, the merger strategy may trigger pseudo-generics' production. We conclude this section by policy recommandations (section 4.2).

4.1 Comparison of the duopoly and the pseudo-duopoly regimes:

Excluding mergers, we determine under which conditions the brand-name firm produces pseudo-generics, and the market, as a consequence, becomes pseudo-duopolistic. To find these conditions, we compare brand-name firm's duopoly and pseudo-duopoly profits:

Proposition 2 *For a high product differentiation ($\alpha > \overline{\alpha}(\beta,\gamma)$), the brand-name laboratory produces pseudo-generics.*

Proof. See appendix J. Moreover,by hypothesis $\alpha < \beta$, thus

$$\text{If } \overline{\alpha}(\beta,\gamma) > \beta \text{ then } \Pi_b^{D*} > \Pi_b^{P*} \text{ for } \alpha \in \left[\frac{\gamma}{2}, \beta\right[, \tag{27}$$

$$\text{If } \overline{\alpha}(\beta,\gamma) < \beta \text{ then } \begin{cases} \Pi_b^{D*} > \Pi_b^{P*} \text{ for } \alpha \in \left[\frac{\gamma}{2}, \overline{\alpha}(\beta,\gamma)\right]. \\ \Pi_b^{P*} > \Pi_b^{D*} \text{ for } \alpha \in \left[\overline{\alpha}(\beta,\gamma), \beta\right[. \end{cases} \tag{28}$$

■

Remark 7 *When $\alpha \in \left[\frac{\gamma}{2}, A(\beta,\gamma)\right[$, the pseudo-duopoly can not exist and thus the duopoly takes place. Finally, for low product differentiation ($\gamma > 2\alpha$), the generic producer is excluded from the industry and the brand-name firm monopolizes the market.*

The own-price effect for the brand-name product is lower than the own-price effect for the generic product, but very low relative to the cross-price effect. The firm has an incentive to produce pseudo-generic goods because the increase in profits earned on pseudo-generic products compensates for the decrease in profits earned on brand-name drugs. This is because of the weakness of the cross-price effect.

Summary of the market structure without merger strategy:

If $\beta < \overline{\alpha}(\beta,\gamma)$:

α values	$0 \ldots\ldots \frac{\gamma}{2}$	$\frac{\gamma}{2} \ldots\ldots \beta$
Market structure	Monoproduct monopoly	Duopoly

If $\beta > \overline{\alpha}(\beta,\gamma)$:

α values	$0 \ldots\ldots \frac{\gamma}{2}$	$\frac{\gamma}{2} \ldots\ldots \overline{\alpha}(\beta,\gamma)$	$\overline{\alpha}(\beta,\gamma) \ldots\ldots \beta$
Market structure	Monoproduct monopoly	Duopoly	Pseudo-duopoly

Remark 8 *Note that $\overline{\overline{\alpha}}(\beta,\gamma) > \overline{\alpha}(\beta,\gamma)$*[13].

[13] $F(\alpha = \overline{\overline{\alpha}}(\beta,\gamma), \beta, \gamma) = -2\gamma^3(\gamma - 2\beta)^2\left(4\beta + 3\gamma + 2\sqrt{3}\beta + \sqrt{3}\gamma\right) < 0$. From appendix I, $\frac{\Pi_b^{D*}}{\Pi_b^{T*}} < 1$, then $\overline{\overline{\alpha}}(\beta,\gamma) > \overline{\alpha}(\beta,\gamma)$.

4.2 "Strategic" merger and competition policy

When a "delayed" merger takes place, the brand-name firm has to produce pseudo-generics during the first period. This firm gives up the monopoly profit it would have earned, by merging at the first stage of the game. Nevertheless, it buys the generic laboratory at a lower cost. We call "strategic" merger a "delayed" merger for which pseudo-generics production has been decided exclusively for a merger motive. This means that the "strategic" merger takes place for a range of demand parameters which excludes the pseudo-generics production when mergers are not considered (see section 4.1). We deduce from this:

Proposition 3 *For* $\alpha \in [\alpha_2(\beta,\gamma), \overline{\alpha}(\beta,\gamma)]$*, if pseudo-generics are produced, it is with the sole purpose of inciting a "strategic" merger.*

Proof. We show that $\alpha_2(\beta,\gamma) < \overline{\alpha}(\beta,\gamma)$ for $\beta > \gamma(\frac{1}{2} + \frac{\sqrt{2}}{2})$ (see appendix K). ■

We saw, in a framework in which mergers strategies are not considered (section 4.1), that the brand-name firm has no incentive to produce pseudo-generic goods when the price-sensitivity for the brand-name product is weak ($\alpha < \overline{\alpha}(\beta,\gamma)$). However, the "delayed" merger possibility modifies this incentive because the brand-name firm produces pseudo-generic goods even if its profits fall. Indeed, the duopoly profit is larger than the pseudo-duopoly profit. Nevertheless, this production allows the brand-name firm to buy the generic firm at a lower price.

This "strategic" merger is preferred to the "standard" merger under the proposition 1 conditions . Obviously, it is also preferred to the status quo (because the "standard" merger dominates the status quo). Thus, the brand-name firm, when $\alpha_2(\beta,\gamma) < \alpha < \overline{\alpha}(\beta,\gamma)$, is ready to sacrifice its duopoly profit for its pseudo-duopoly profit, with the aim of acquiring the generic firm at a lower cost.

From a policy perspective, the absence of consumer surplus analysis in our paper does not allow us to propose recommendations. However, we can anticipate some implications. Obviously, in the absence of efficiency gains, as it is the case in our model, mergers should be blocked. Our model indicates that the pseudo-generics production must be considered by competition authorities as a potential merger signal. More exactly, if demand parameters are such that firms have incentives to produce pseudo-generics when they do not consider mergers ($\alpha > \overline{\alpha}(\beta,\gamma)$), the pseudo-generics production is not a merger signal. We can even consider that if $\overline{\alpha}(\beta,\gamma) < \alpha < \overline{\overline{\alpha}}(\beta,\gamma)$ and that $\delta < \overline{\overline{\delta}}(\alpha,\beta,\gamma)$, the "delayed" merger strategies do not exist. On the other hand, if the demand parameters are such that a "strategic" merger is practicable ($\alpha_2(\beta,\gamma) < \alpha < \overline{\alpha}(\beta,\gamma)$), the pseudo-generics production surely signals a merger. Indeed, no other strategy could justify such a production because pseudo-duopoly profits are less important than duopoly profits.

5 Conclusion

Our paper analyzes mergers on the pharmaceutical market. More exactly, we prove that the pseudo-generics' production may be a means towards a lower cost merger. Indeed, this strategy reduces the purchase price of the generic company. We modelize an industry in which a brand-name firm and a generic firm compete *à la* Cournot. Brand-name and generic goods are considered as imperfect substitutes. This model presents a two-stage non-cooperative game. At each stage, the brand-name firm can purchase its generic competitor. But, at the first stage, the brand-name firm can decide to produce pseudo-generics instead. In this framework, the generic laboratory is a Stackelberg leader on the generic market where two homogeneous goods are available: generic and pseudo-generic products.

Three results emerge. First, without pseudo-generics, the brand-name producer always monopolizes the market. Second, pseudo-generics production may delay the takeover. Indeed, under some conditions, the "delayed" merger dominates the first period merger. Third, the brand-name company can produce pseudo-generics solely to monopolize the industry, even if it is not profitable at first. Indeed, under some market conditions, the pseudo-generics production reduces the brand-name firm's profit, but this firm nevertheless decides to produce pseudo-generics in order to reduce the purchase price of the generic manufacturer. Two conditions must be satisfied to insure the profitability of this strategy: the loss due to the pseudo-generics production over a period, must not be too significant. This is the case if the products are differentiated enough *i.e* if the pseudo-duopoly is not too unfavourable. Moreover, the discounted value of the gain relative to the reduction in the purchase price must be high enough. But, the higher product differentiation is, the lower the rate of discount can be.

From a policy perspective, the absence of studies on consumer surplus in our paper does not allow us to propose recommendations. However, we can anticipate some implications. Obviously, in the absence of efficiency gains, as it is the case in our model, mergers should be blocked. First, our model indicates the pseudo-generics production must be considered by competition policy as a potential merger signal. On the other hand, the competition authorities must not neglect the fact that the pseudo-generics entry increases consumer surplus because of the increased competition. Moreover, the consumer surplus analysis is interesting for another reason. Kong and Seldon (2004) show that barriers to entry are created due to the pseudo-generics production. This anti-competitive practice reduces consumer surplus. In our study, generics are present in the industry before pseudo-generics. The latter facilitate mergers and we may think they decrease consumer surplus. Therefore, such a consumer surplus analysis could result in the prohibition of pseudo-generic products.

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Appendix A (duopoly):

$$\Pi_g^D = p_g q_g = (1 - \beta q_g - \gamma q_b) q_g. \quad (A.1)$$

$$\Pi_b^D = p_b q_b = (1 - \alpha q_b - \gamma q_g) q_b. \quad (A.2)$$

Competition *à la* Cournot between the two firms:
The first order condition (FOC) of the generic laboratory requires that:

$$\begin{aligned} \frac{\partial \Pi_g^D}{\partial q_g} &= (1 - \beta\, q_g - \gamma\, q_b) - \beta q_g = 0 \quad (A.3) \\ \Longrightarrow \; q_g &= -\frac{1}{2\beta}\left(\gamma q_b - 1\right). \end{aligned}$$

The FOC of the brand-name laboratory requires that:

$$\begin{aligned} \frac{\partial \Pi_b^D}{\partial q_b} &= (1 - \alpha q_b - \gamma\, q_g) - \alpha q_b = 0 \quad (A.4) \\ \Longrightarrow \; q_b &= -\frac{1}{2\alpha}\left(\gamma q_g - 1\right). \end{aligned}$$

Appendix B (multiproduct monopoly):

$$\begin{aligned} \Pi_b^M &= p_b q_b + p_g q_g \quad (B.1) \\ &= (1 - \alpha q_b - \gamma\, q_g) q_b + (1 - \beta\, q_g) - \gamma\, q_b) q_g. \end{aligned}$$

The monopoly chooses simultaneously its two quantities in order to maximize its profit function.

The FOC 1 gives us the response function in brand-name output:

$$\frac{\partial \Pi_b^M}{\partial q_b} = -2\alpha q_b + 1 - 2\gamma q_g = 0 \;\Rightarrow q_b = -\frac{1}{2\alpha}\left(2\gamma q_g - 1\right). \quad (B.2)$$

The FOC 2 gives us the response function in pseudo-generic output:

$$\frac{\partial \Pi_b^M}{\partial q_g} = -2\gamma q_b - 2\beta q_g + 1 = 0 \;\Rightarrow q_g = -\frac{1}{2\beta}\left(2\gamma q_b - 1\right). \quad (B.3)$$

Appendix C (monoproduct monopoly):

If $\alpha < \gamma$, the monopoly produces only the brand-name product. Therefore, it maximizes the following profit function:

$$\Pi_b^{MM} = p_b q_b = (1 - \alpha q_b) q_b. \quad (C.1)$$

The FOC give us the equilibrium brand-name output:

$$\frac{\partial \Pi_b^{MM}}{\partial q_b} = 0 \Rightarrow 1 - 2\alpha q_b = 0 \Rightarrow q_b^{MM*} = \frac{1}{2\alpha}. \quad (C.2)$$

Appendix D (merger incentive):

Case 1: $\alpha \in \left[\frac{\gamma}{2}, \gamma\right[$:

$$\begin{aligned} F(\alpha,\beta,\gamma) &= \Pi_b^{MM*} - \Pi_g^{D*} - \Pi_b^{D*} \\ &= -\frac{1}{4}\frac{(2\alpha-\gamma)(8\beta\alpha^2 - 12\alpha\beta\gamma + 2\alpha\gamma^2 + \gamma^3)}{\alpha(4\alpha\beta-\gamma^2)^2}. \end{aligned} \tag{D.1}$$

$sign(F(\alpha,\beta,\gamma)) \neq sign(f(\alpha,\beta,\gamma))$ with:

$$f(\alpha,\beta,\gamma) = 8\beta\alpha^2 + (-12\beta\gamma + 2\gamma^2)\alpha + \gamma^3. \tag{D.2}$$

$f(\alpha,\beta,\gamma)$ is a trinomial in α. Its determinant is:

$$4\gamma^2(18\beta-\gamma)(2\beta-\gamma) > 0 \tag{D.3}$$

$f(\alpha,\beta,\gamma)$ admits two roots:

$$\begin{aligned} \widehat{\alpha}(\beta,\gamma) &= \frac{1}{8}\frac{6\beta\gamma - \gamma^2 - \sqrt{36\beta\gamma^2 - 20\beta\gamma^3 + \gamma^4}}{\beta}, \\ \widetilde{\alpha}(\beta,\gamma) &= \frac{1}{8}\frac{6\beta\gamma - \gamma^2 + \sqrt{36\beta\gamma^2 - 20\beta\gamma^3 + \gamma^4}}{\beta}. \end{aligned} \tag{D.4}$$

As $6\beta\gamma - \gamma^2 > 0$ and $36\beta\gamma^2 - 20\beta\gamma^3 + \gamma^4 > 0$,

$$\begin{aligned} 6\beta\gamma - \gamma^2 - \sqrt{36\beta\gamma^2 - 20\beta\gamma^3 + \gamma^4} &> 0 \Leftrightarrow \\ (6\beta\gamma-\gamma^2)^2 - 36\beta^2\gamma^2 + 20\beta\gamma^3 - \gamma^4 &> 0. \end{aligned} \tag{D.5}$$

Since $(6\beta\gamma-\gamma^2)^2 - 36\beta^2\gamma^2 + 20\beta\gamma^3 - \gamma^4 = 8\beta\gamma^3 > 0$,
then $\widehat{\alpha}(\beta,\gamma) > 0$.

$$\widehat{\alpha}(\beta,\gamma) - \frac{\gamma}{2} = \frac{1}{8}\frac{2\beta\gamma - \gamma^2 - \sqrt{\gamma^2(18\beta-\gamma)(2\beta-\gamma)}}{\beta}. \tag{D.6}$$

As $2\beta\gamma - \gamma^2 > 0$ et $\gamma^2(18\beta-\gamma)(2\beta-\gamma) > 0$,
and that $(2\beta\gamma-\gamma^2)^2 - \gamma^2(18\beta-\gamma)(2\beta-\gamma) = -16\beta\gamma^2\,(2\beta-\gamma) < 0$,
then $\widehat{\alpha}(\beta,\gamma) < \frac{\gamma}{2}$.

Moreover, $\widetilde{\alpha}(\beta,\gamma) > \widehat{\alpha}(\beta,\gamma)$, therefore, $\widetilde{\alpha}(\beta,\gamma) > 0$.

$$\widetilde{\alpha}(\beta,\gamma) - \gamma = -\frac{1}{8}\frac{(2\beta\gamma + \gamma^2 - \sqrt{\gamma^2(18\beta-\gamma)(2\beta-\gamma)})}{\beta}. \tag{D.7}$$

As $2\beta\gamma + \gamma^2 > 0$ et $\gamma^2(18\beta-\gamma)(2\beta-\gamma)$,
and that $(2\beta\gamma+\gamma^2)^2 - \gamma^2(18\beta-\gamma)(2\beta-\gamma) = -8\beta\gamma^2\,(4\beta-3\gamma) < 0$,
then $\widetilde{\alpha}(\beta,\gamma) > \gamma$.

For $\alpha \in \left[\frac{\gamma}{2}, \gamma\right[$, then $\alpha \in \left[\widehat{\alpha}(\beta,\gamma), \widetilde{\alpha}(\beta,\gamma)\right]$, therefore $f(\alpha,\beta,\gamma) \leq 0$. Subsequently, $F(\alpha,\beta,\gamma) \geq 0$. The merger is thus always profitable.

Case 2: $\alpha \in [\gamma, \beta[$:

$$F(\alpha,\beta,\gamma,\delta) = \Pi_b^{M*} - \Pi_g^{D*} - \Pi_b^{D*} \tag{D.8}$$
$$= \frac{1}{4}\frac{\gamma^2\left(4\alpha\beta^2 + 5\beta\gamma^2 - 16\alpha\beta\gamma - 2\gamma^3 + 4\alpha^2\beta + 5\alpha\gamma^2\right)}{\left(\alpha\beta - \gamma^2\right)\left(4\alpha\beta - \gamma^2\right)^2}.$$

$sign(F(\alpha,\beta,\gamma)) = sign(n(\alpha,\beta,\gamma))$ with :

$$n(\alpha,\beta,\gamma) = 4\alpha^2\beta + \left(4\beta^2 - 16\gamma\beta + 5\gamma^2\right)\alpha + 5\beta\,\gamma^2 - 2\gamma^3. \tag{D.9}$$

$n(\alpha,\beta,\gamma)$ is a trinomial in α. Its determinant is :

$$\Delta(\beta,\gamma) = \left(4\,\beta^2 - 28\gamma\beta + 25\gamma^2\right)\left(2\beta - \gamma\right)^2. \tag{D.10}$$

$sign(\Delta(\beta,\gamma)) = sign(4\,\beta^2 - 28\gamma\beta + 25\gamma^2)$.
Let $\Delta'(\beta,\gamma)$ be the determinant of this trinomial in β. $\Delta'(\beta,\gamma) = 384\gamma^2 > 0$.
This trinomial admits two roots:

$$\beta_1(\gamma) = \frac{7}{2}\gamma - \sqrt{6}\gamma > \gamma, \tag{D.11}$$
$$\beta_2(\gamma) = \frac{7}{2}\gamma + \sqrt{6}\gamma.$$

Therefore, for $\beta \in [\beta_1(\gamma), \beta_2(\gamma)]$, $n(\alpha,\beta,\gamma) > 0$. Thus, the merger is profitable for these parameters values.

For $\beta \in [\gamma, \beta_1(\gamma)]$ and for $\beta \in [\beta_2(\gamma), +\infty]$, $n(\alpha,\beta,\gamma)$ admits two roots $\alpha^*(\beta,\gamma)$ et $\alpha^{**}(\beta,\gamma)$:

$$\alpha^*(\beta,\gamma) = \frac{-4\beta^2 - 16\gamma + 5\beta\gamma^2}{8\beta} - \frac{\sqrt{16\beta^4 - 128\beta^3\gamma + 216\beta^2\gamma^2 - 128\beta\gamma^3 + 25\gamma^4}}{8\beta}, \tag{D.12}$$
$$\alpha^{**}(\beta,\gamma) = \frac{-4\beta^2 - 16\gamma + 5\beta\gamma^2}{8\beta} + \frac{\sqrt{16\beta^4 - 128\beta^3\gamma + 216\beta^2\gamma^2 - 128\beta\gamma^3 + 25\gamma^4}}{8\beta}.$$

We deduct 3 cases:

If $\alpha \in \left]\gamma, \alpha^*(\beta,\gamma)\right]$, then $n(\alpha,\beta,\gamma) \geq 0$. Therefore $F(\alpha,\beta,\gamma) \geq 0$,
If $\alpha \in \left]\alpha^*(\beta,\gamma), \alpha^{**}(\beta,\gamma)\right[$, then $n(\alpha,\beta,\gamma) < 0$. Therefore $F(\alpha,\beta,\gamma) < 0$,
If $\alpha \in \left[\alpha^{**}(\beta,\gamma), +\infty\right[$, then $n(\alpha,\beta,\gamma) \geq 0$. Therefore $F(\alpha,\beta,\gamma) \geq 0$.

Note:

$$\mu^*(\beta,\gamma) = \alpha^*(\beta,\gamma) - \gamma = \frac{1}{8}\frac{-4\beta^2 + 8\beta\gamma - 5\gamma^2 - \sqrt{(4\beta^2 - 28\beta\gamma + 25\gamma^2)(2\beta-\gamma)^2}}{\beta}. \tag{D.13}$$

Note:

$$\mu^{**}(\beta,\gamma) = \alpha^{**}(\beta,\gamma) - \gamma = \frac{1}{8}\frac{-4\beta^2 + 8\beta\gamma - 5\gamma^2 + \sqrt{(4\beta^2 - 28\beta\gamma + 25\gamma^2)(2\beta-\gamma)^2}}{\beta}. \tag{D.14}$$

$4\beta^2 - 28\beta\gamma + 25\gamma^2 > 0$ for $\beta \in [\gamma, \beta_1(\gamma)]$ and for $\beta \in [\beta_2(\gamma), +\infty]$.
$-4\beta^2 + 8\beta\gamma - 5\gamma^2$ is a trinomial in β. Its determinant is $\Delta"(\beta,\gamma) = -16\gamma^2 < 0$.

Therefore $\mu^*(\beta,\gamma) < 0$.
Moreover,

$$\mu^*(\beta,\gamma).\mu^{**}(\beta,\gamma) = 16\beta\gamma(4\beta - 3\gamma)(\beta-\gamma) > 0. \tag{D.15}$$

Therefore $\mu^{**}(\beta,\gamma) < 0$.
At last, we can affirm that $\alpha^*(\beta,\gamma) < \gamma$ and that $\alpha^{**}(\beta,\gamma) < \gamma$. As we assume that $\alpha \in [\gamma, \beta[$, *i.e* $\alpha > 0$, then $n(\alpha,\beta,\gamma) > 0$. Subsequently, $F(\alpha,\beta,\gamma) > 0$ in this second case.

Appendix E (pseudo-duopoly case):

$$\Pi_g^T = (1 - \beta\,(q_g + q_{pg}) - \gamma\, q_b)q_g. \tag{E.1}$$

$$\Pi_b^T = (1 - \alpha q_b - \gamma(q_g + q_{pg}))q_b + (1 - \beta(q_g + q_{pg}) - \gamma q_b)q_{pg}. \tag{E.2}$$

$$\begin{aligned} \frac{\partial \Pi_b^T}{\partial q_{pg}} &= -2\gamma q_b - \beta q_{pg} + 1 - \beta(q_g + q_{pg}) = 0 \\ &\implies q_{pg} = RF(q_b, q_g) = -\frac{1}{2}\frac{2\gamma q_b - 1 + \beta q_g}{\beta}. \end{aligned} \tag{E.3}$$

Substituting $RF(q_b, q_g)$ in the two profit functions, we obtain :

$$\Pi_g^T = \frac{1}{2}(1 - \beta q_g)q_g. \tag{E.4}$$

$$\Pi_b^T = \frac{1}{4}\frac{(\gamma^2 - \alpha\beta)q_b^2 + (\beta-\gamma)q_b + \beta^2 q_g^2 - 2\beta q_g + 1}{\beta}. \tag{E.5}$$

Appendix F (cost and gain study):

1) $c(\beta)$ analysis:

Note :

$$c(\beta) = \frac{3}{16\beta}. \tag{F.1}$$

$c(\beta) > 0$, $\forall\beta$

2) $g(\alpha,\beta,\gamma,\delta)$ analysis:

$$g(\alpha,\beta,\gamma,\delta) = \frac{1}{8}\frac{\left(4\alpha\beta - \gamma^2\right)^2\delta - 8\beta^2\left(2\alpha - \gamma\right)^2}{\beta(4\alpha\beta - \gamma^2)^2(\delta - 1)}. \tag{F.2}$$

$$g(\alpha,\beta,\gamma,\delta)=0 \text{ for } \overline{\delta}(\alpha,\beta,\gamma)=\frac{8\beta^2\left(2\alpha-\gamma\right)^2}{\left(4\alpha\beta-\gamma^2\right)^2}. \tag{F.3}$$

Note that we study $g(\alpha,\beta,\gamma,\delta)$ only for $\delta\in[0,1[$.

- Variation study of $g(\alpha,\beta,\gamma,\delta)$ in δ:

$$\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta}=\frac{1}{8}\frac{16\beta^2\alpha^2+\left(-32\beta^2\gamma+8\beta\gamma^2\right)\alpha-\gamma^4+8\beta^2\gamma^2}{\left(4\alpha\beta-\gamma^2\right)^2\beta\left(\delta-1\right)^2}. \tag{F.4}$$

$sign(\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta})=sign(16\beta^2\alpha^2+\left(-32\beta^2\gamma+8\beta\gamma^2\right)\alpha-\gamma^4+8\beta^2\gamma^2)$
$16\beta^2\alpha^2+\left(-32\beta^2\gamma+8\beta\gamma^2\right)\alpha-\gamma^4+8\beta^2\gamma^2$ is a trinomial in α. Its determinant is $128\beta^2\gamma^2\left(2\beta-\gamma\right)^2>0$.

This trinomial admits two roots:

$$\begin{aligned}\alpha_1(\beta,\gamma) &= \frac{\beta\gamma-\frac{1}{4}\gamma^2-\frac{1}{4}\sqrt{8\beta^2\gamma^2-8\beta\gamma^3+2\gamma^4}}{\beta}, \\ \alpha_2(\beta,\gamma) &= \frac{\beta\gamma-\frac{1}{4}\gamma^2+\frac{1}{4}\sqrt{8\beta^2\gamma^2-8\beta\gamma^3+2\gamma^4}}{\beta}.\end{aligned} \tag{F.5}$$

Consequently, three cases are possible:

$$\begin{aligned}\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} &> 0 \text{ for } \alpha\in\left]-\infty,\alpha_1(\beta,\gamma)\right], \\ \frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} &< 0 \text{ for } \alpha\in\left[\alpha_1(\beta,\gamma),\alpha_2(\beta,\gamma)\right], \\ \frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} &> 0 \text{ for } \alpha\left[\alpha_2(\beta,\gamma),+\infty\right[.\end{aligned}$$

Remark:

$$\begin{aligned}&\alpha_1(\beta,\gamma)-A(\alpha,\beta,\gamma) \\ =\ &\frac{1}{4}\frac{-4\beta\gamma+3\gamma^2-\sqrt{2}\gamma\left(2\beta-\gamma\right)}{\beta}<0.\end{aligned} \tag{F.6}$$

Therefore, the zone with $\alpha\in\left]-\infty,\alpha_1(\beta,\gamma)\right]$ is excluded from the study zone of the "delayed" mergers because the pseudo-generic can not be produced when $\alpha<A(\alpha,\beta,\gamma)$.

$$\begin{aligned}&\alpha_2(\beta,\gamma)-A(\alpha,\beta,\gamma) \\ =\ &\frac{1}{4}\frac{-4\beta\gamma+3\gamma^2+\sqrt{2}\gamma\left(2\beta-\gamma\right)}{\beta}>0, \text{ si } \beta>\gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}.\end{aligned} \tag{F.7}$$

Therefore, if $\beta>\gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$,

$$\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta}>0 \text{ for } \alpha\in\left[A(\alpha,\beta,\gamma),+\infty\right[.$$

And if $\beta < \gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$,

$$\begin{aligned}\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} &< 0 \text{ for } \alpha \in [A(\alpha,\beta,\gamma),\alpha_2(\beta,\gamma)]\,,\\ \frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} &> 0 \text{ for } \alpha \in [\alpha_2(\beta,\gamma),+\infty[\,.\end{aligned}$$

By hypothesis, $\beta > \alpha$, it is thus necessary to classify β, $\alpha_2(\beta,\gamma)$, et $A(\beta,\gamma)$.
$A(\beta,\gamma) - \beta < 0\ \forall\alpha,\beta,\gamma$.
Moreover, $\alpha_2(\beta,\gamma) - \beta = -\frac{1}{4}\frac{(2\beta-\gamma)(2\beta-\gamma-\sqrt{2}\gamma)}{\beta} < 0$ if $\beta > \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$.

Then:
- If $\beta > \gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$, $\alpha_2(\beta,\gamma) < A(\alpha,\beta,\gamma)$. As $\gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}} > \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$, then $\beta > \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$ and $\alpha_2(\beta,\gamma) < A(\beta,\gamma) < \beta$.
- If $\beta < \gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$, $\alpha_2(\beta,\gamma) > A(\alpha,\beta,\gamma)$. Two cases are possible:
If $\beta > \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$, then $A(\beta,\gamma) < \alpha_2(\beta,\gamma) < \beta$.
If $\beta < \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$, then $\alpha_2(\beta,\gamma) < A(\beta,\gamma) < \beta$.

To summarize:
- If $\beta > \gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$, then $\alpha_2(\beta,\gamma) < A(\beta,\gamma) < \beta$.
- If $\gamma(\frac{1}{2}+\frac{\sqrt{2}}{2}) < \beta < \gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$, then $A(\beta,\gamma) < \alpha_2(\beta,\gamma) < \beta$.
- If $\beta < \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$, then $A(\beta,\gamma) < \beta < \alpha_2(\beta,\gamma)$.

We show that:
If $\beta > \gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$, then $\alpha_2(\beta,\gamma) < A(\beta,\gamma) < \beta$ and

$$\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} > 0 \text{ for } \alpha \in [A(\beta,\gamma),\beta[\,. \tag{F.8}$$

If $\gamma(\frac{1}{2}+\frac{\sqrt{2}}{2}) < \beta < \gamma\frac{3-\sqrt{2}}{4-2\sqrt{2}}$, then $A(\beta,\gamma) < \alpha_2(\beta,\gamma) < \beta$ and

$$\begin{aligned}\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} &< 0 \text{ for } \alpha \in [A(\beta,\gamma),\alpha_2(\beta,\gamma)]\,.\\ \frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} &> 0 \text{ for } \alpha \in [\alpha_2(\beta,\gamma),\beta[\,.\end{aligned} \tag{F.9}$$

If $\beta < \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$, then $A(\beta,\gamma) < \beta < \alpha_2(\beta,\gamma)$ and

$$\frac{\partial g(\alpha,\beta,\gamma,\delta)}{\partial\delta} < 0 \text{ for } \alpha \in [A(\beta,\gamma),\beta]\,. \tag{F.10}$$

- Discontinuity study of $g(\alpha,\beta,\gamma,\delta)$:

In the first case:
If $\alpha < \alpha_2(\beta,\gamma)$ then $\underset{\delta\to 1^-}{lim}\ g(\alpha,\beta,\gamma,\delta) = -\infty$.
In the second case:
If $\alpha > \alpha_2(\beta,\gamma)$ then $\underset{\delta\to 1^-}{lim} g(\alpha,\beta,\gamma,\delta) = +\infty$.

- Let us study $\overline{\delta}(\alpha, \beta, \gamma)$ in α:

$\overline{\delta} = 1$ for $\alpha_1(\beta, \gamma)$ et $\alpha_2(\beta, \gamma)$ (see (76) et (77)).

Three cases are possible:

$$\begin{aligned}\overline{\delta}(\alpha, \beta, \gamma) &> 1 \text{ for } \alpha \in \left]-\infty, \alpha_1(\beta, \gamma)\right], \\ \overline{\delta}(\alpha, \beta, \gamma) &< 1 \text{ for } \alpha \in \left[\alpha_1(\beta, \gamma), \alpha_2(\beta, \gamma)\right], \\ \overline{\delta}(\alpha, \beta, \gamma) &> 1 \text{ for } \alpha \left[\alpha_2(\beta, \gamma), +\infty\right[.\end{aligned}$$

As $\alpha_1(\beta, \gamma) < A(\beta, \gamma)$, only two cases are possible:

$$\begin{aligned}\overline{\delta}(\alpha, \beta, \gamma) &< 1 \text{ for } \alpha \in \left[A(\beta, \gamma), \alpha_2(\beta, \gamma)\right], \\ \overline{\delta}(\alpha, \beta, \gamma) &> 1 \text{ for } \alpha \in \left[\alpha_2(\beta, \gamma), +\infty\right[.\end{aligned}$$

Therefore:

$$\begin{aligned}\overline{\delta}(\alpha, \beta, \gamma) &< 1 \text{ for } \alpha < \alpha_2(\beta, \gamma), \\ \overline{\delta}(\alpha, \beta, \gamma) &> 1 \text{ for } \alpha > \alpha_2(\beta, \gamma).\end{aligned}$$

and $\overline{\delta}(\alpha_2(\beta, \gamma), \beta, \gamma) = 1$. Another root $\alpha_1(\beta, \gamma)$ exists but $\alpha_1(\beta, \gamma) < A(\beta, \gamma)$. Since the pseudo-generic can not be produced if $\alpha > A(\beta, \gamma)$, $\alpha_1(\beta, \gamma)$ is not interesting for our "delayed" merger problem.

- Conclusion about $g(\alpha, \beta, \gamma, \delta)$ (for $\delta \in [0; 1[$):

In the case where $\alpha < \alpha_2(\beta, \gamma)$, then $g(\alpha, \beta, \gamma, \delta)$ is decreasing in δ, and $g(\alpha, \beta, \gamma, \delta) = 0$ for $\delta = \overline{\delta}(\alpha, \beta, \gamma)$. Note that $\overline{\delta}(\alpha, \beta, \gamma) < 1$ for $\alpha < \alpha_2(\beta, \gamma)$. When δ is near to 1, $g(\alpha, \beta, \gamma, \delta)$ tends towards $-\infty$. Therefore $g(\alpha, \beta, \gamma, \delta) \geq 0$ for $\delta \in \left[0, \overline{\delta}(\alpha, \beta, \gamma)\right]$ and $g(\alpha, \beta, \gamma, \delta) < 0$ for $\delta \in \left[\overline{\delta}(\alpha, \beta, \gamma), 1\right[$.

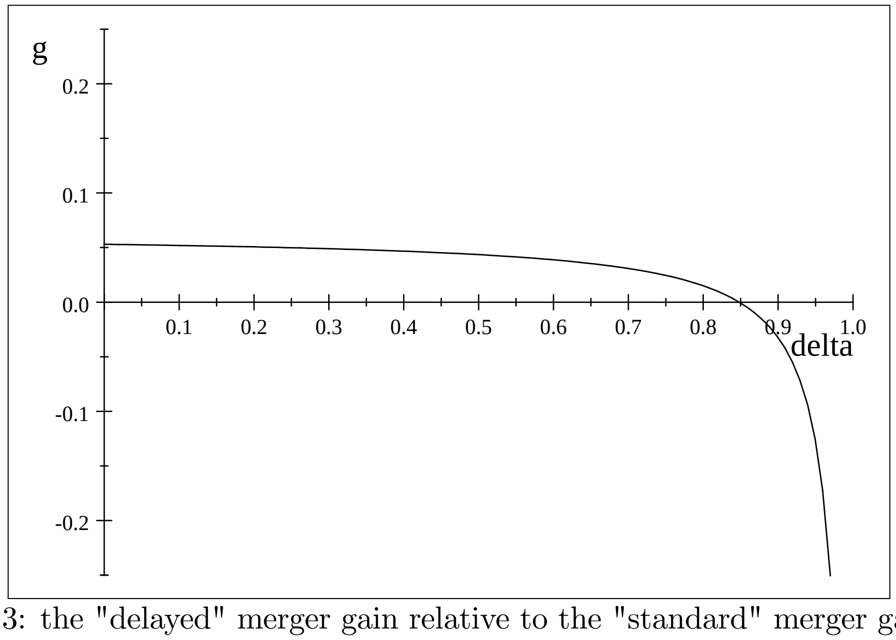

Figure 3: the "delayed" merger gain relative to the "standard" merger gain (for $\alpha < \alpha_2(\beta, \gamma)$)

In the case where $\alpha > \alpha_2(\beta,\gamma)$, then $g(\alpha,\beta,\gamma,\delta)$ is increasing in δ, and $g(\alpha,\beta,\gamma,\delta) = 0$ for $\delta = \overline{\delta}(\alpha,\beta,\gamma)$. Note that $\overline{\delta}(\alpha,\beta,\gamma) > 1$ for $\alpha > \alpha_2(\beta,\gamma)$. When δ is near to 1, $g(\alpha,\beta,\gamma,\delta)$ tends towards $+\infty$. Therefore $g(\alpha,\beta,\gamma,\delta) > 0$ for $\delta \in [0,1]$.

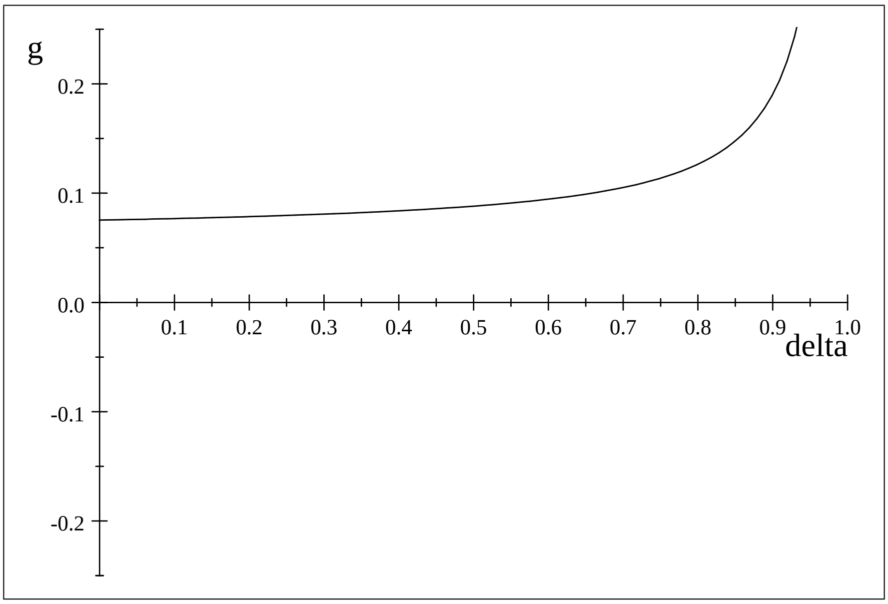

Figure 4: the "delayed" merger gain relative to the "standard" merger gain (for $\alpha > \alpha_2(\beta,\gamma)$)

Appendix G (study of the difference between cost and gain associated to "delayed" merger relative to "standard" merger):

$$\begin{aligned} S(\alpha,\beta,\gamma,\delta) &= g(\alpha,\beta,\gamma,\delta) - c(\beta) \\ &= \frac{\left(16\alpha^2\beta^2 - 8\alpha\beta\gamma^2 + \gamma^4\right)\delta}{16\left(\gamma^2 - 4\alpha\beta\right)^2 \beta\,(1-\delta)} \\ &\quad + \frac{\left(16\alpha^2\beta^2 - 64\alpha\beta^2\gamma + 24\alpha\beta\gamma^2 + 16\beta^2\gamma^2 - 3\gamma^4\right)}{16\left(\gamma^2 - 4\alpha\beta\right)^2 \beta\,(1-\delta)}. \end{aligned} \tag{G.1}$$

As $c(\beta)$ is independent from δ, $S(\alpha,\beta,\gamma,\delta)$ variation is the same than $g(\alpha,\beta,\gamma,\delta)$ variation.

$S(\alpha,\beta,\gamma,\delta) = 0$ for :

$$\overline{\overline{\delta}}(\alpha,\beta,\gamma) = \frac{\left(-16\beta^2\right)\alpha^2 + \left(64\beta^2\gamma - 24\beta\gamma^2\right)\alpha + \left(3\gamma^4 - 16\beta^2\gamma^2\right)}{\left(4\alpha\beta - \gamma^2\right)^2}. \tag{G.2}$$

$\overline{\overline{\delta}}(\alpha,\beta,\gamma) = 1$ for $\alpha_1(\beta,\gamma)$ and $\alpha_2(\beta,\gamma)$.

Same manner as for $\overline{\delta}(\alpha,\beta,\gamma)$, we deduce that $\overline{\overline{\delta}}(\alpha,\beta,\gamma) > 1$ for $\alpha < \alpha_2(\beta,\gamma)$ and $\overline{\overline{\delta}}(\alpha,\beta,\gamma) < 1$ for $\alpha > \alpha_2(\beta,\gamma)$.

- Conclusion:

If $\alpha < \alpha_2(\beta,\gamma)$, then $S(\alpha,\beta,\gamma,\delta)$ is decreasing in δ and tends towards $-\infty$ when δ tends towards 1. As $S(\alpha,\beta,\gamma,\delta)$ does not admit a root in this case, we can affirm that $S(\alpha,\beta,\gamma,\delta) < 0$. This means that the "standard" merger will be always preferred to the "delayed" merger for these parameters values. The "standard" merger being always advantageous, one will thus have always this type of merger at the Nash equilibrium.

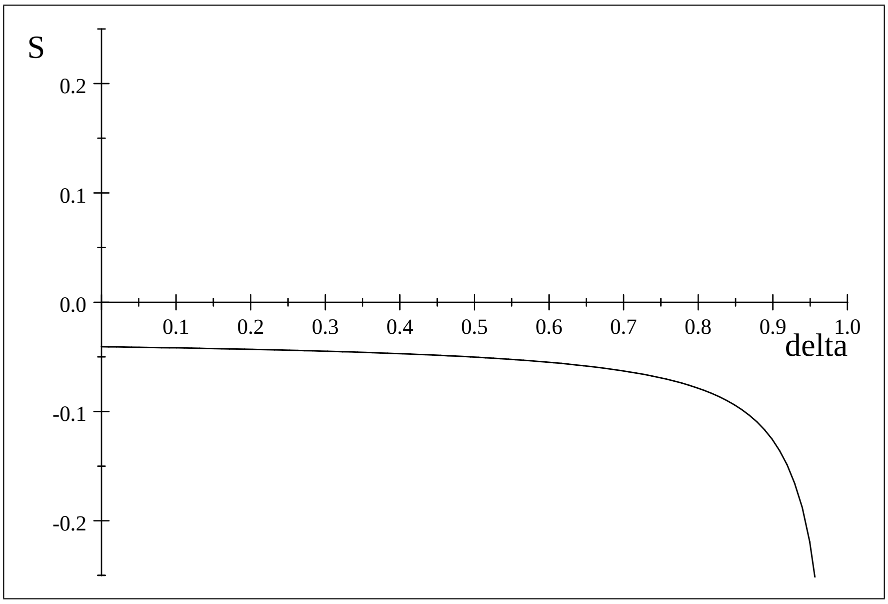

Figure 5: incentive difference between the "strategic" merger gain and the "delayed" merger gain (for $\alpha < \alpha_2(\beta,\gamma)$)

If $\alpha > \alpha_2(\beta,\gamma)$ (possible only for $\beta > \gamma(\frac{1}{2}+\frac{\sqrt{2}}{2})$), then $S(\alpha,\beta,\gamma,\delta)$ is increasing with δ and when δ tends towards 1, $S(\alpha,\beta,\gamma,\delta)$ tends towards $+\infty$. $S(\alpha,\beta,\gamma,\delta) = 0$ for $\delta = \overline{\overline{\delta}}(\alpha,\beta,\gamma)$. We deduce that $S(\alpha,\beta,\gamma,\delta) \geq 0$ for $\delta \in \left[\overline{\overline{\delta}}(\alpha,\beta,\gamma), 1\right[$. Therefore, the "delayed" merger is the Nash equilibrium of the game. On the contrary, if $\delta \in \left[0, \overline{\overline{\delta}}(\alpha,\beta,\gamma)\right[$, the "standard" merger is the Nash equilibrium.

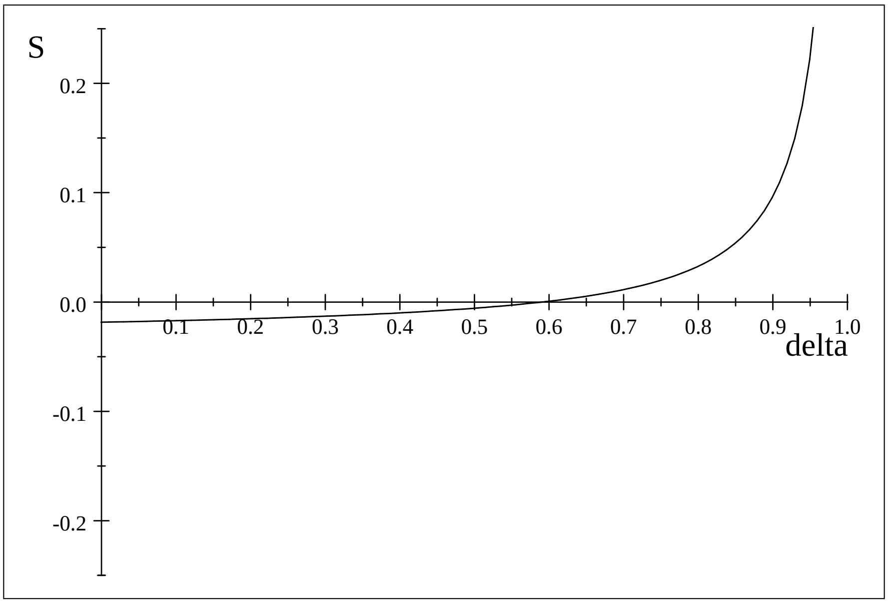

Figure 6: incentive difference between the "delayed" merger gain and the "standard" merger gain (for $\alpha > \alpha_2(\beta, \gamma)$)

Two Nash equilibria are possible according to the values of demand and actualization parameters:

If $\overline{\alpha} > \alpha > \alpha_2(\beta, \gamma)$ (only possible if $\beta > \gamma(\frac{1}{2} + \frac{\sqrt{2}}{2})$) and that $\delta \in \left[\overline{\overline{\delta}}(\alpha, \beta, \gamma), 1\right[$, the brand-name firm produces pseudo-generics in a first time then purchases the generic firm in a second time. At first, the generic firm changes its production because of the presence of pseudo-generics, then accepts to be sold at a price equal to its pseudo-duopoly profit.

Appendix H (static comparative analysis)

$$\overline{\overline{\delta}}(\alpha, \beta, \gamma) = \frac{\left(-16\beta^2\right)\alpha^2 + \left(64\beta^2\gamma - 24\beta\gamma^2\right)\alpha + \left(3\gamma^4 - 16\beta^2\gamma^2\right)}{\left(4\alpha\beta - \gamma^2\right)^2}. \tag{H.1}$$

$\frac{\partial \overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial \alpha} = \frac{64\gamma(\gamma - 2\alpha)(\gamma - 2\beta)\beta^2}{(\gamma^2 - 4\alpha\beta)^3} < 0$ for $\alpha > \frac{\gamma}{2}$. As $\alpha_2(\beta, \gamma) > \frac{\gamma}{2}$ and that $\overline{\overline{\delta}}(\alpha, \beta, \gamma)$ is defined for $\alpha > \alpha_2(\beta, \gamma)$, then $\frac{\partial \overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial \alpha} < 0$ for the interesting values of parameters (concerning "delayed" mergers).

$\frac{\partial \overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial \beta} = -32\beta \frac{\gamma^2(\gamma - 2\alpha)^2}{(\gamma^2 - 4\alpha\beta)^3} > 0$.

$\frac{\partial \overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial \gamma} = 32 \frac{\beta^2(\gamma - 2\alpha)}{(\gamma^2 - 4\alpha\beta)^3}\left(\gamma^2 - 4\alpha\gamma + 4\alpha\beta\right) > 0$ for $\alpha > \frac{\gamma}{2}$. As $\alpha_2(\beta, \gamma) > \frac{\gamma}{2}$ and that $\overline{\overline{\delta}}(\alpha, \beta, \gamma)$ is defined for $\alpha > \alpha_2(\beta, \gamma)$, then $\frac{\partial \overline{\overline{\delta}}(\alpha,\beta,\gamma)}{\partial \gamma} < 0$ for the interesting values of parameters (concerning "delayed" mergers).

Appendix I (discounted rate neutrality):

We showed that $\frac{\partial(\overline{\overline{\delta}}(\alpha,\beta,\gamma))}{\partial \alpha} < 0$ for $\alpha > \alpha_2(\beta, \gamma)$. $\overline{\overline{\delta}}(\alpha, \beta, \gamma) = 0$ for $\underline{\underline{\alpha}}(\beta, \gamma) = -\frac{1}{4\beta}\left(-2\sqrt{3}\gamma^2 + 3\gamma^2 - 8\beta\gamma + 4\sqrt{3}\beta\gamma\right)$ and $\overline{\overline{\alpha}}(\beta, \gamma) = -\frac{1}{4\beta}\left(2\sqrt{3}\gamma^2 + 3\gamma^2 - 8\beta\gamma - 4\sqrt{3}\beta\gamma\right)$.

$\underline{\underline{\alpha}}(\beta, \gamma) > 0$ and $\overline{\overline{\alpha}}(\beta, \gamma) > 0$

Moreover, $\alpha_2(\beta,\gamma) - \underline{\underline{\alpha}}(\beta,\gamma) = \frac{1}{4}\gamma\frac{-\beta\left(2\sqrt{2}-4\sqrt{3}+4\right)+\gamma\left(\sqrt{2}-2\sqrt{3}+2\right)}{\beta} > 0$ and $\alpha_2(\beta,\gamma) - \overline{\overline{\alpha}}(\beta,\gamma) = -\frac{1}{4}\gamma\frac{\beta\left(2\sqrt{2}+4\sqrt{3}+4\right)-\gamma\left(\sqrt{2}+2\sqrt{3}+2\right)}{\beta} < 0$.

Therefore, in the interesting zone, *i.e* $\alpha > \alpha_2(\beta,\gamma)$, $\overline{\overline{\delta}}(\alpha,\beta,\gamma)$ is decreasing in α and $\overline{\overline{\delta}}(\alpha,\beta,\gamma) = 0$ for $\alpha = \overline{\overline{\alpha}}(\beta,\gamma)$.

Appendix J (comparison of the two structures):

$$\text{Note } \frac{\Pi_b^{D*}}{\Pi_b^{T*}} = \frac{16\alpha\beta(2\alpha\beta-\gamma)^2(\alpha\beta-\gamma^2)}{(4\alpha\beta-\gamma^2)^2(\alpha\beta+4\beta^2-8\gamma\beta+3\gamma^2)}, \tag{J.1}$$

$$\text{with } \alpha > A(\beta,\gamma) > \frac{\gamma}{2} \begin{cases} \text{if } \frac{\Pi_b^{D*}}{\Pi_b^{T*}} > 1 \text{ then } \Pi_b^{D*} > \Pi_b^{T*}. \\ \text{if } \frac{\Pi_b^{D*}}{\Pi_b^{T*}} < 1 \text{ then } \Pi_b^{D*} < \Pi_b^{T*}. \end{cases} \tag{J.2}$$

$$\text{Note } \begin{cases} N(\alpha,\beta,\gamma) = 16\alpha\beta(2\alpha\beta-\gamma)^2(\alpha\beta-\gamma^2). \\ D(\alpha,\beta,\gamma) = (4\alpha\beta-\gamma^2)^2(\alpha\beta+4\beta^2-8\gamma\beta+3\gamma^2). \end{cases} \tag{J.3}$$

$$\text{If } \frac{\Pi_b^{D*}}{\Pi_b^{T*}} > 1 \text{ (respectively} < 1\text{) then } N(\alpha,\beta,\gamma) - D(\alpha,\beta,\gamma) > 0 \text{ (respectively} < 0\text{).} \tag{J.4}$$

$$\begin{aligned} F(\alpha,\beta,\gamma) &= N(\alpha,\beta,\gamma) - D(\alpha,\beta,\gamma) \\ &= -32\alpha\beta^3\gamma^2 + 64\alpha^2\beta^3\gamma - 24\alpha^2\beta^2\gamma^2 \\ &\quad +7\alpha\,\beta\gamma^4 - 16\alpha^3\beta^3 - 4\gamma^4\beta^2 + 8\gamma^5\beta - 3\gamma^6. \end{aligned} \tag{J.5}$$

Study of $F(\alpha,\beta,\gamma)$:

$$\begin{aligned} F(\alpha,\beta,\gamma)_{|\alpha=A(\beta,\gamma)} &= 2\gamma^3(\beta-\gamma)(2\beta-\gamma)(-9\gamma+16\beta). \\ \frac{\partial F(\alpha,\beta,\gamma)}{\partial\alpha} &= \left(128\alpha\gamma - 48\alpha^2 - 32\gamma^2\right)\beta^3 \\ &\quad + \left(-48\alpha\gamma^2\right)\beta^2 + \left(7\gamma^4\right)\beta. \\ \frac{\partial F(\alpha,\beta,\gamma)}{\partial\alpha}_{|\alpha=A(\beta,\gamma)} &= \gamma^2\beta\left(32\beta^2 - 32\gamma\beta + 7\gamma^2\right). \\ \frac{\partial^2 F(\alpha,\beta,\gamma)}{\partial\alpha^2} &= 128\beta^3\gamma - 48\beta^2\gamma^2 - 96\beta^3\alpha. \\ \frac{\partial^2 F(\alpha,\beta,\gamma)}{\partial\alpha^2}_{|\alpha=A(\beta,\gamma)} &= -16\beta^2\gamma(-3\gamma+4\beta). \\ \frac{\partial^3 F(\alpha,\beta,\gamma)}{\partial\alpha^3} &= -96\beta^3. \end{aligned} \tag{J.6}$$

$\frac{\partial^3 F(\alpha,\beta,\gamma)}{\partial\alpha^3} < 0$ therefore $\frac{\partial^2 F(\alpha,\beta,\gamma)}{\partial\alpha^2}$ is decreasing in α.

For $\alpha = A(\beta,\gamma)$, $\frac{\partial^2 F(\alpha,\beta,\gamma)}{\partial\alpha^2} < 0$, therefore $F(\alpha,\beta,\gamma)$ is concave for $\alpha \in [A(\beta,\gamma), +\infty[$.

In addition, for $\alpha = A(\beta,\gamma)$, $F(\alpha,\beta,\gamma) > 0$ and $\frac{\partial F(\alpha,\beta,\gamma)}{\partial\alpha} > 0$. Therefore, for $\alpha = A(\beta,\gamma)$, $F(\alpha,\beta,\gamma)$ is positive and increasing in α. As it is concave for $\alpha \in [A(\beta,\gamma), +\infty[$, $F(\alpha,\beta,\gamma) > 0$ for $\alpha \in [A(\beta,\gamma), \overline{\alpha}(\beta,\gamma)[$, and $F(\overline{\alpha}(\beta,\gamma),\beta,\gamma) = 0$.

Moreover, pseudo-generics production (viability conditions of the pseudo-duopoly are not respected) is not possible when $\alpha \in \left[\frac{\gamma}{2}, A(\beta,\gamma)\right[$.

Two zones thus appear:

$$\begin{aligned} \alpha &\in \left[\frac{\gamma}{2}, \overline{\alpha}(\beta,\gamma)\right[\text{ where } \Pi_b^{D*} > \Pi_b^{T*}, \\ \alpha &\in [\overline{\alpha}(\beta,\gamma), +\infty[\text{ where } \Pi_b^{T*} > \Pi_b^{D*}. \end{aligned}$$

Appendix K ("strategic" merger):

We showed in appendix 8 that $F(\alpha,\beta,\gamma)$ is quasi concave for $\alpha > A(\beta,\gamma)$. $F(\alpha,\beta,\gamma)$ admits a maximum for $\alpha_{\max}(\beta,\gamma) = -\frac{1}{12\beta}\gamma\left(-16\beta + 6\gamma + \sqrt{160\beta^2 + 57\gamma^2 - 192\beta\gamma}\right)$ with $A(\beta,\gamma) < \alpha_{\max}(\beta,\gamma) < \overline{\alpha}(\beta,\gamma)$.

$\alpha_2(\beta,\gamma) > 0$. If $\alpha_2(\beta,\gamma) - \alpha_{\max}(\beta,\gamma) < 0$ then $\alpha_2(\beta,\gamma) < \alpha_{\max}(\beta,\gamma)$.

Let us show that $\alpha_2(\beta,\gamma) - \alpha_{\max}(\beta,\gamma) < 0$.

$\alpha_2(\beta,\gamma) - \alpha_{\max}(\beta,\gamma) = \frac{1}{12\beta}\left(\gamma\sqrt{160\beta^2 - 192\beta\gamma + 57\gamma^2} + 3\gamma^2 - 4\beta\gamma + 3\sqrt{2}\gamma(\gamma - 2\beta)\right)$

$\frac{\partial(\alpha_2(\beta,\gamma) - \alpha_{\max}(\beta,\gamma))}{\partial\beta} > 0$ for $\alpha > A(\beta,\gamma)$.

The "delayed" mergers imply that $\beta > \gamma(\frac{1}{2} + \frac{\sqrt{2}}{2})$ (see proposition 2).

If $\beta > \gamma(\frac{3-\sqrt{2}}{4-2\sqrt{2}})$, then $\alpha_2(\beta,\gamma) < A(\beta,\gamma) < \overline{\alpha}(\beta,\gamma)$.

If $\gamma(\frac{1}{2} + \frac{\sqrt{2}}{2}) < \beta < \gamma(\frac{3-\sqrt{2}}{4-2\sqrt{2}})$, $\alpha_2(\beta,\gamma) - \alpha_{\max}(\beta,\gamma) < 0$ because, for $\beta = \gamma(\frac{3-\sqrt{2}}{4-2\sqrt{2}})$, $\alpha_2(\beta,\gamma) - \alpha_{\max}(\beta,\gamma) < 0$ and because $\frac{\partial(\alpha_2(\beta,\gamma) - \alpha_{\max}(\beta,\gamma))}{\partial\beta} > 0$ for $\alpha > A(\beta,\gamma)$.

Therefore $\alpha_2(\beta,\gamma) < \overline{\alpha}(\beta,\gamma)$ for $\beta > \gamma(\frac{1}{2} + \frac{\sqrt{2}}{2})$.