Mixed Bundling and Mergers

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Abstract

Does bundling trigger mergers? We observe mergers between firms belonging to independent industries. These mergers enable firms to bundle. Indeed, many telephone firms, internet access providers or cable TV operators merge. Thus, the merged firms can provide bundles. Therefore, the question is the following: can bundling strategies allowed by a two-market merger create an incentive to merge? We consider two horizontally differentiated markets. The correlation of reservation prices is the sole link between these two markets. In this framework, we show that bundling strategies create incentives to form multi-markets firms. Merger decisions are endogenous in our model.

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1 Introduction

Does bundling trigger mergers? We observe mergers between firms belonging to independent industries. These mergers enable firms to bundle. Indeed, many telephone firms, internet access providers or cable TV operators merge. Thus, the merged firms can provide bundles. Therefore, the question is the following: can bundling strategies allowed by a two-market merger create an incentive to merge? We consider two horizontally differentiated markets. The correlation of reservation prices is the sole link between these two markets. In this framework, we show that bundling strategies create incentives to form multi-markets firms. Merger decisions are endogenous in our model.

In many sectors, packages are developed. In the telecommunications sector, "double play", "triple play" and "quadruple play" packages[1] appeared. This empirical reality leads to the idea that firms supplying only one type of good can have an incentive to merge in order to sell several products together. Particularly, we observe mergers allowing firms to bundle. In the USA in 2005, the telecommunications firm SBC bought out AT&T for 16 billion dollars and took its name. The same year, Verizon, its main competitor, took over MCI for 8,4 billion dollars. Through mergers, these firms can be present in several markets simultaneously. In this framework, one of the main merger objectives is probably the bundling strategy. In the same way, in the European market, the merger between France Télécom and Wanadoo, well as the merger between Deutsch Telekom and T-Online[2] are relevant illustrations of bundling incentives.

Several reasons may explain merger incentives. A great deal of literature exists in this field. Different kinds of merger motives are suggested. Mainly, there can be either financial reasons, or economics ones. The maximization of shareholder value, (Jensen and Meckling, 1976) is one of them. Another can be the potential growth caused by the takeover of an indebted firm that cannot invest. A less indebted firm with a low potential growth can buy it out and finance its investments. Attaining a critical mass (Bradley, Desai and Kim, 1983), realizing efficiency gains in production costs (Molnar, 2006) or in fixed costs (Rodrigues, 2001) are other reasons to merge. But a fundamental economic explanation is the decrease in competition[3] (Eckbo, 1983 , Kamien and Zang, 1990). However, a merger between firms from different markets does not decrease competition. There is probably another motive for this type of merger. Indeed, can bundling strategies create an incentive to merge between firms from two different markets? To answer this question, we remove from our model any data other than bundling strategies which can trigger a merger.

To better understand how bundling strategies can create merger incentives, one must know some principles concerning these strategies. Bundling refers to the practice of selling two or several goods at a unique price. When a firm sells its goods both separately and bundled in a package, this firm follows a mixed bundling strategy. When a firm commits

[1] These bundles can combine Internet access, fix and mobile phones, as TV.

[2] In 2004, the French operator France Télécom buys out Wanadoo, its Internet subsidiary whose shares it owned. The latter are converted into France Telecom shares. Now, France Télécom supplies triple-play packages consisting of phone, Internet and TV. In 2005, ditto for Deutsch Telekom and its subsidiary T-online.

[3] Even so, note that Salant, Switzer and Reynolds (1983) show that there is an incentive to merge only if more than 80 per cent of the industry is involved in the merger. But the competition decrease has a positive effect on merger incentives.

to supply only the bundle, it follows a pure bundling strategy. The economic literature on bundling isolates several effects. One of the main effects is price discrimination. Bundling allows to sort consumers according to their willingness to pay. This characteristic is analyzed by Adams and Yellen (1976) for a monopoly producing two goods. This analysis deals with specific cases[4]. They show the bundling strategy is generally the optimal strategy. Nalebuff (2004) underlines another consequence of bundling strategies. Indeed, a two-market monopolistic firm can deter entry by bundling if the entrant can enter only one market. In this framework, he shows that pure bundling is optimal. Considering a competition environment, a second effect appeared besides the sorting effect of bundling: a competition effect. Anderson and Leruth (1993) analyze bundling in a complementary-goods duopoly. In their view, independent pricing is a dominant strategy in the commitment case. Economides (1993), in the same framework, shows that mixed bundling is the Nash equilibrium. However, firms make lower profits than when adopting an independent pricing strategy.

Reisinger (2006) also studies a duopoly producing two types of horizontally differentiated goods. But, he excludes the complementarity assumption. The correlation of the reservation prices is expressed by the correlation of consumers' location on each market. He shows there are two effects created by bundling. The "sorting effect" is the first wellknown effect. The second effect is the "business-stealing effect". It is due to bundle competition. He shows that firms have an incentive to adopt a mixed bundling strategy. Nonetheless, the effect on profits is ambiguous. If the correlation of reservation prices is negative, then the competition effect dominates and the bundling strategy lowers profits. Firms are in a prisoner's dilemma situation. On the other hand, if the correlation of reservation prices is positive, then the sorting effect allows firms to make higher profits.

We use the model of Reisinger (2006) in order to analyze the impact of bundling on merger incentives. Therefore, we consider two horizontally differentiated markets. The link between these two markets is the correlation of consumers' locations. However, we study the effect of bundling on merger incentives. In order not to neglect mergers interactions, we endogenize merger decisions. In this sense, our study is closely linked to the endogenous merger literature. There are various types of endogenous merger models. Some of them seek to explain mechanisms preventing mergers as the "insider's dilemma" already evoked in the exogenous merger model of Stigler (1950). Kamien et Zang (1990, 1993) do the same but they add auction mechanisms to take into account firms acquisitions processes. We also care about the "insider's dilemma" but without any auction mechanism. Indeed, we are not interested in surplus sharing rule. We rather want to determine if mergers can be achieved somehow or other. On the other hand, we deal with other characteristics of endogenous merger literature. Indeed, these characteristics concern taking all firms' combinations into consideration. For instance, some endogenous merger models allow to reveal mergers interactions (Nilssen et Sorgard, 1998). More particularly, some models attempt to emphasize on preemptive mergers phenomena (Fridolfsson et Stennek, 2005, Brito 2003, Matsushima, 2001). Finally, other models focus on some phenomena as merger waves. Besides merger waves are explained by Fauli-Oller (2000) or Nilssen and Sorgard (1998). As the same type of merger interactions is possible in our framework, we build a merger game based upon Nilssen and Sorgard (1998).

[4]Schmalensee (1984) shows their results are robust to a bivariate normal distribution. As for them, Mc Afee, McMillan and Whinston (1989) generalize these results to almost all distributions.

We model two horizontally differentiated markets. Initially, four firms are present. Two firms produce the first type of good and the two others supply the other one. In their respective markets, firms compete in prices. We build an endogenous merger game and we assume that monopolization is illegal. The merger game is a sequential and non cooperative one. First, we exclude the post merger bundling strategy. Second, we remove this assumption in order to analyze the effect of bundling strategy on merger incentives.

In a basic model in which bundling is not considered, we find there is no incentive to merge. But, once a merger is achieved, we show there is an incentive to adopt a mixed bundling strategy. Otherwise, the bundling strategy triggers a merger wave. Moreover, we show that relative to the correlation of reservation prices, two types of mergers are achieved. Furthermore, while Reisinger (2006) shows there is a prisoner's dilemma, we show that the different types of mergers allow to remove this dilemma. Finally, from a welfare point of view, we show that bundling is harmful but to a lesser extent than in Reisinger (2006). The following section presents the basic model. The section 3 introduces the bundling strategy. The section 4 gives the equilibrium of the game and the social welfare analysis. Finally, the last section presents some concluding remarks.

2 Basic model

Throughout this section, we exclude bundling strategies. We start with the assumptions of the competition game. Next, a merging game is defined. Finally, we solve the game in order to establish the benchmark before introducing the bundling strategy in the next section.

2.1 Assumptions

We consider an industry composed of four firms. Two firms produce the good A at the marginal cost c_A and two others the good B at the marginal cost c_B. In order not to introduce bias[5] in our bundling analysis, we assume production costs are linear. Each type of goods is horizontally differentiated. Therefore, the product space for each good is taken to be the unit-circonference of a circle. The product variants are then the locations of the firms on each circle. According to the type and the location of their output, firms are named either A_i or B_j with $i, j = 1, 2$. The firm A_1 produces the good A and is located at 0 on circle A while the firm A_2 produces the same good[6] but is located at $\frac{1}{2}$. There is a continuum of consumers and without loss of generality, we normalize its total mass to 1. Consumers' locations on both circles are $x = (x_A, x_B)^T$. Every consumer has a unit demand for each good and purchases each good independently of the other. Thus, there is no complementarity between the products. This allows us to focus on the pure strategic effect of bundling. Firms compete in prices on each market. Their prices are denoted by p_A^i and p_B^i. Thus, consumers can choose between four product combinations. They can buy either the good A to firm A_1 and the good B to firm B_1, *i.e.* $(A1B1)$, or the good A to firm A_2 and the good B to firm B_2, *i.e.* $(A2B2)$, or the good A to firm A_1

[5] In this model, bundling must not be incited by efficiency gains for instance.

[6] Note that we deliberately choose to place firms at locations 0 and $\frac{1}{2}$, but without loss of generality. Indeed, if we placed firms more closely, results would be qualitatively the same. They would just be shifted relative to δ.

and the good B to firm B_2, *i.e.* ($A1B2$), or the good A to firm A_2 and the good B to firm B_1, *i.e.* ($A2B1$).

For instance, a customer located at $0 \leq x_A, x_B \leq 1/2$, buying the good A from firm A_1 and the good B from firm B_2 has an indirect utility of:

$$V(x_A, x_B) = K_A - p_A^1 - t_A(x_A)^2 + K_B - p_B^2 - t_B(1/2 - x_B)^2. \quad (1)$$

K_A and K_B are the surpluses from consumption (gross of prices and transportation costs) of goods A and B. If the two markets (or the two goods) are denoted by $k = A, B$. We note t_k the transportation cost associated with circle k. Without loss of generality[7], we assume $t_A > t_B > 0$. Therefore, the consumer reservation price for the variant i of the good k, R_k^i, is $K_k - t_k(d_i)^2$, where d_i is the shortest arc length between the consumer's location and firm i on circle k. It is also assumed that K_k is sufficiently large, so that in each price equilibrium all consumers buy both goods. Concerning reservation prices, they can be linked to consumers' locations. Indeed, the joint distribution function of reservation values $G(R_A^i \mid R_B^i)$ and so, the correlation between reservation prices for the good from location i on the two markets can be deduced from the joint distribution function of consumer location $F(x_A \mid x_B)$. Like Reisinger (2006), we give a structure to this distribution function. It is a simple function that still captures the main point of expressing different correlations. We assume that if a consumer is located at x_A on circle A, then it is located at

$$x_B = \begin{cases} x_A + \delta & \text{if } x_A + \delta \leq 1 \\ x_A + \delta - 1 & \text{if } x_A + \delta > 1 \end{cases}$$

on circle B, where $0 \leq \delta \leq 1/2$. This means a δ-shift of all consumers on circle B. So a δ of 0 expresses a reservation price correlation of 1. Adopting this simple structure, correlations of reservation values can be obtained easily by altering δ. The correlation coefficient $\rho[R_A, R_B](\delta) = \frac{Cov[R_A, R_B](\delta)}{\sigma(R_A)\sigma(R_B)}$ is given[8] by $1 - 30\delta^2 + 60\delta^3 - 30\delta^4$. By way of illustration, note that for small δ values, if a consumer has a high reservation price for the good of firm A_1, then he has a high reservation value for the good of firm B_1. If a consumer has a high reservation price for the good of firm A_2, then he has a high reservation value for the good of firm B_2. Conversely, a high value of δ implies that consumers have very different reservation prices for the two goods sold at the same location on each circle. Now, we define the merger game in which the four firms are involved.

2.2 Merger game

We assume that monopolization is illegal[9]. Thus, potential mergers necessarily involve firms from two different markets. We build a two-stage game. At the first stage, firms choose either to merge or not. At the second one, they compete in prices. The previous section already presented these stage assumptions. We describe now the first stage. Each firm A_i can choose to merge with the firm with the same location in the other market. We

[7] The limit cases $t_B \to t_A$ and $t_B \to 0$ are studied in section 4.

[8] The proof is given in Reisinger (2006).

[9] There is always an incentive to monopolize the market A or the market B as long as reservation prices of consumers are sufficiently high. But this is detrimental for consumers and generally, authorities forbid monopolization.

call this type of merger "homogeneous merger". Each firm A_i can also choose to merge with the firm located at the opposite in the other market. In this case, the merger is called "heterogeneous merger". Finally, each firm A_i can choose not to merge (see figure 1 below).

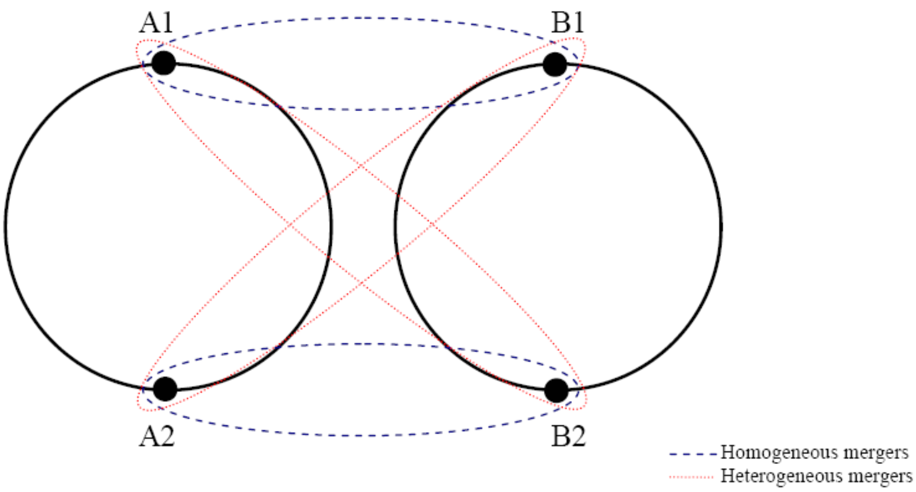

Figure 1: homogeneous and heterogeneous mergers

Firms take sequential and non-cooperative decisions. Without loss of generality, we assume decisions are taken by firms of the market A. Results are exactly similar if decisions are taken by firms of market B. We assume also that firm A_1 takes first his merger decision but the results are the same if firm A_2 chooses first. In this paper, we look for subgame perfect Nash equilibria (SPNE). Therefore, we solve the game backward. Thus, we solve competition subgames then we solve the merger game. Concerning mergers, we do not define any profit sharing rule. We exclusively focus on a question: is the merger profit higher than the pre-merger profit sum of firms involving in the merger? Indeed, if such is the case, there is necessarily a profit sharing rule that gives an incentive to merge. On the other hand, in the same way as the "strategic motives" consideration (Nilssen et Sorgard, 1998), we take account of interactions between merger decisions. To illustrate, two-firm merger expectation can either incite or not another two-firm merger. Therefore, merger decisions are endogenous in this model. At the first stage, both firms A_1 and A_2 choose to merge or not to merge with firms of the market B. Thus, we present this game in figure 2 below.

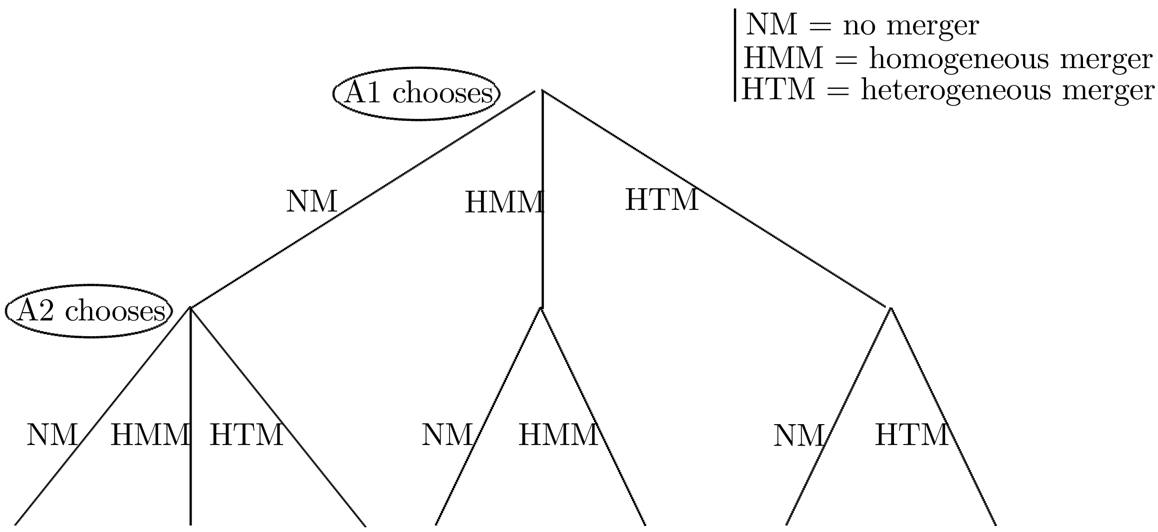

Figure 2: game tree

Now, we solve the game without considering bundling strategy. This constitutes the benchmark.

2.3 Benchmark: mergers and independent pricing:

By symmetry, we deduce from the game tree that there are four possible outcomes. There may be either two homogeneous mergers, or two heterogeneous mergers, or an only one homogeneous merger, or an only one heterogeneous merger, or finally no merger. We determine prices and profits of the different game's outcomes. We note Π_{A_i} (respectively Π_{B_j}) the profit of the single firm A_i (respectively B_j). In every case, equilibrium prices are given by[10]:

$$p_A^* = c_A + \frac{1}{4}t_A, \qquad (2)$$
$$p_B^* = c_B + \frac{1}{4}t_B.$$

First, we consider the case where there is no merger. Thus, there are two monoproduct firms in each market A and B competing in prices. The equilibrium profits are given by:

$$\Pi_{A_i}^* = \frac{1}{8}t_A, \qquad (3)$$
$$\Pi_{B_j}^* = \frac{1}{8}t_B.$$

We note $\Pi_{A_iB_j}^z$ the profit of the merged firm A_iB_j where $z = 1, 2$ is the number of mergers. Whether there is one or two homogeneous mergers, the equilibrium profits are given by:

$$\forall i = j, \quad \Pi_{A_iB_j}^{2*} = \frac{1}{8}t_A + \frac{1}{8}t_B, \quad \text{or} \qquad (4)$$
$$\forall i = j, \quad \Pi_{A_iB_j}^{1*} = \frac{1}{8}t_A + \frac{1}{8}t_B, \quad \Pi_{A_{-i}}^* = \frac{1}{8}t_A, \quad \Pi_{B_{-j}}^* = \frac{1}{8}t_B.$$

[10]In the independent pricing case, the computation of the equilibrium is the same as in the standard model of Salop (1979). Indeed, we model two independent and horizontally differentiated markets.

Whether there is one or two heterogeneous mergers, the equilibrium profits are given by:

$$\forall i \neq j, \quad \Pi^{2*}_{A_iB_j} = \frac{1}{8}t_A + \frac{1}{8}t_B, \quad \text{or} \tag{5}$$
$$\forall i \neq j, \quad \Pi^{1*}_{A_iB_j} = \frac{1}{8}t_A + \frac{1}{8}t_B, \quad \Pi^*_{A_{-i}} = \frac{1}{8}t_A, \quad \Pi^*_{B_{-j}} = \frac{1}{8}t_B.$$

It is easy to show the following proposition:

Proposition 1 *If bundling strategy is not considered, then there is no incentive to merge. Indeed,* $\forall z = 1, 2$*:*

$$\Pi^z_{A_iB_j} = \Pi_{A_i} + \Pi_{B_j}, \quad \forall i, j = 1, 2. \tag{6}$$

When they cannot provide bundles, the monoproduct firms are indifferent between merge or not. Since markets are independent, there is no competition effect due to mergers. Therefore, there is no incentive to merge[11]. After a merger, prices are unchanged and the global profit of a merger is merely the profit sum of the merging firms. In this case of indifference, we assume that firms choose not to merge. After the benchmark analysis, the following section considers the case where merged firms can follow mixed bundling strategy. Thus, we focus on pure effects of bundling in the competition game, as on their impacts on incentives to merge.

3 Mergers and mixed bundling

In this section, we introduce mixed-bundling strategy[12]. Indeed, if two firms merge, they can supply a bundle composed of both types of goods. Now, a merged firm A_iB_j $\forall i, j = 1, 2$ can offer a package composed of good A_i and good B_j. This bundle is denoted by $(ABij)$ and its price is p^{ij}_{AB}. Mixed bundling may enable firms to attract marginal consumers by lowering the bundle price. Indeed, a consumer who buys the goods from different firms in the independent pricing case can then prefer to buy the two goods from the same firm. The desutility when purchasing its non prefered variant of a good is balanced by a lower price. We show that results depend on correlation of reservation prices. In this section, we analyze game outcomes. Now, mixed bundling strategy is available. Thus, we study competition in each outcome. But, we exclude the non-merged outcome because bundling is not possible in this case and so, profits are already computed in the previous section. For simplicity, we just analyze the merger waves outcomes, *i.e.* the two-merger cases. We will explain why it is sufficient to solve the game in section 4. First, we study the case of homogeneous two-firm merger. This one corresponds to Reisinger's model (2006). Next, a sub-section is devoted to the case of heterogeneous two-firm mergers.

3.1 Bundling and homogeneous mergers

We assume firms A_i merge with firms B_j, $\forall i = j$, with $i, j = 1, 2$. In this configuration, customers choose between four product combinations. They can either buy the bundle of

[11] Here, we model no other mechanism which is able to create a merger incentive. For instance, considering synergies could create an incentive to merge.

[12] We exclude the pure bundling strategy since firms have no incentive to manage without an additional discrimination tool, given that firms are in competition.

firm A_1B_1, *i.e.* $(AB11)$ at price p_{AB}^{11}, or buy the bundle of firm A_2B_2, *i.e.* $(AB22)$ at price p_{AB}^{22}. They can also purchase either the good A from firm A_1B_1 and the good B from firm A_2B_2, *i.e.* the product combination $(A1B2)$ at price $p_A^1 + p_B^2$, or the good A from firm A_2B_2 and the good B from firm A_1B_1, *i.e.* the consumption option $(A2B1)$ at price $p_A^2 + p_B^1$. We want to determine prices and profits when firms adopt a mixed bundling strategy but this is interesting only if firms have an incentive to bundle. Thus, we use the lemma of Reisinger (2006):

Lemma 1 *If $\delta > 0$, i.e $\rho < 1$, then in equilibrium, homogeneous merged firms follow a mixed-bundling strategy.*

Proof. See Reisinger (2006) ■

In the benchmark, there is no merger incentive. Merger incentives are only due to bundling strategies. The lemma shows that if there are two mergers, each firm follows a mixed bundling strategy. In the same manner, every firm is incited to merge because merger incentives and bundling ones are exactly the same here. In this case, two homogeneous mergers are achieved. The proof is exactly the same as the proof of the lemma given by Reisinger (2006). For this reason, we only study the two-homogeneous merger outcome.

Lemma 2 *Respectively for $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$, $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3}\frac{t_B}{6t_A}$, and $\delta > \frac{1}{3}\frac{t_B}{6t_A}$, equilibrium profits are given by:*

$$\Pi_{A_iB_j}^{2*} = \frac{1}{8}(t_A + t_B) + \frac{2}{9}\delta^2 \frac{t_A t_B}{t_A + t_B}, \tag{7}$$

$$\Pi_{A_iB_j}^{2*} = \frac{1}{8}(t_A + t_B) + \frac{t_A t_B(4(t_A + t_B) - 2\delta(6t_A + t_B) - 4\delta^2 t_A)}{2(t_A - t_B)^2}, \tag{8}$$

$$\Pi_{A_iB_j}^{2*} = \frac{1}{8}t_A - \frac{7}{72}t_B. \tag{9}$$

Proof. See Reisinger (2006) ■

First, we assume δ is small, *i.e* the correlation of reservation prices is positive. In this case, there are four consumption combinations: $(AB11)$, $(A1B2)$, $(AB22)$ and $(A2B1)$. The figure 3 below presents this demand configuration (i):

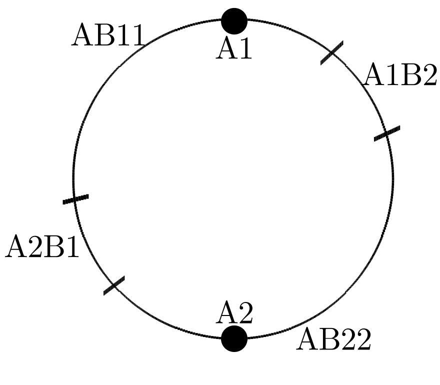

Figure 3: strong correlation

Second, we assume δ is high. In this case, there are six consumption options: $(A1B2)$, $(AB11)$, $(AB22)$, $(A2B1)$, $(AB22)$ and $(AB11)$. The figure 4 below presents this demand configuration (ii):

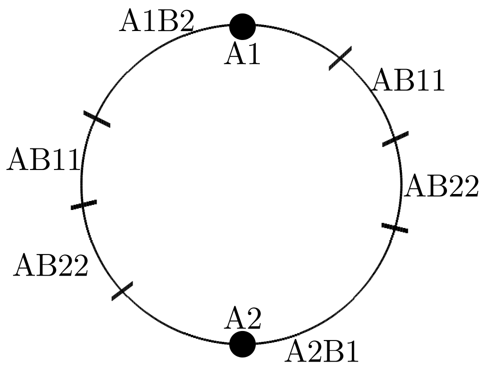

Figure 4: weak correlation

Intuitively, the profit levels explanation is the following. In every case, firms compete *à la* Bertrand on each horizontally differentiated good. But here, they can use bundling. Thus, bundle demands appear. When the correlation is high (δ small), firms compete their own separate goods by lowering their bundle prices. Therefore, they have no incentive to decrease their bundle prices relative to the prices sum of the separate goods in the independent pricing strategy. Furthermore, bundling allows to sort consumers and raises profits. Moreover, this effect increases when correlation decreases. Indeed, each firm can raise its separate goods prices. Thus, profits increase. On the other hand, the correlation decrease makes consumers more and more indifferent to the two bundles. Consequently, for a sufficiently weak correlation ($\delta > \delta_1^{HM}$), each bundle competes directly with the rival's one. Firms must lower their bundle prices in order to maintain their market shares. However, by integrating this new competition for this level of correlation (δ_1^{HM}), the profit maximization does not give an equilibrium. Hence the necessity to compute a touching equilibrium (Economides 1984) between this correlation threshold and the one for which the equilibrium becomes stable (δ_2^{HM}). This implies a linear drop in prices to keep the initial demand structure (i). This new effect dominates the positive sorting effect. The prices decrease entails a drop in profits, and, for a sufficiently high correlation, a prisoner dilemma concerning the bundling strategy decision. We computed the equilibrium of the competiton game following a homogeneous merger wave. However, results can differ when mergers are heterogeneous. Afterwards, we study this game outcome.

3.2 Bundling and heterogeneous mergers

We assume firms A_i merge with firms B_j, $\forall i \neq j$, with $i, j = 1, 2$. In this configuration, customers choose between four product combinations. They can either buy the bundle of firm A_1B_2, *i.e.* $(AB12)$ at price p_{AB}^{12}, or buy the bundle of firm A_2B_1, *i.e.* $(AB21)$ at price p_{AB}^{21}. They also can either purchase the good A to firm A_1B_2 and the good B to firm A_2B_1, *i.e.* the product combination $(A1B1)$ at price $p_A^1 + p_B^1$, or purchase the good A to firm A_2B_1 and the good B to firm A_1B_2, *i.e.* the consumption option $(A2B2)$ at price $p_A^2 + p_B^2$. We want to determine prices and profits when firms follow a mixed-bundling strategy but this is interesting only if firms have an incentive to bundle. Thus, we establish this lemma:

Lemma 3 *If $\delta > 0$, i.e $\rho < 1$, then in equilibrium, heterogeneous merged firms follow a mixed-bundling strategy.*

Proof. see appendix 7.1 ■

In the benchmark, there is no merger incentive. Merger incentives are only due to bundling strategies. The lemma 3 shows that if there are two mergers, each firm adopts a mixed bundling strategy. In the same manner, every firm is incited to merge because merger incentives and bundling ones are exactly the same here. In this case, two heterogeneous mergers are achieved. The proof is exactly the same as the proof of the lemma 3. For this reason, we only study the two-heterogeneous merger outcome.

First, equilibrium demand configurations will be described[13]. The latter depend on δ. We assume δ is small, that is to say ρ is high, *i.e* the correlation is positive. In this case, there are six consumption options: $(A1B1)$, $(AB12)$, $(AB21)$, $(A2B2)$, $(AB21)$, and $(AB12)$. Below, the figure 5 presents this demand configuration (iii):

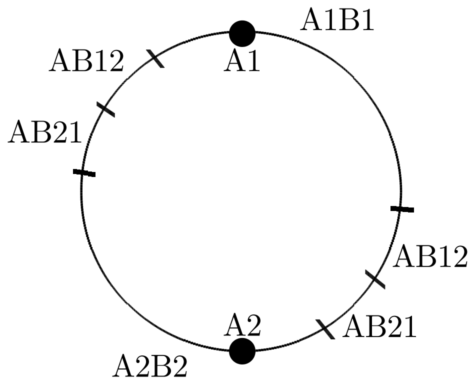

Figure 5: strong correlation

Second, we assume δ is high. In this case, there are no longer four consumption options left: $(AB12)$, $(A2B2)$, $(AB21)$, and $(A1B1)$. The figure 6 below presents this demand configuration (iv):

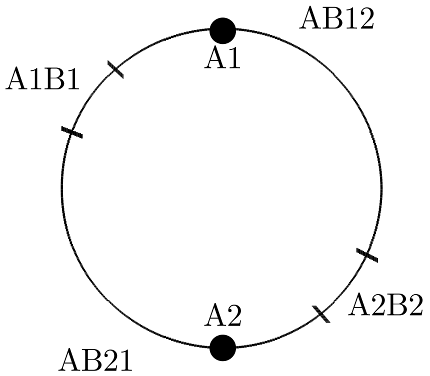

Figure 6: weak correlation

The difference between the two demand configurations is the following: on the right half of the circle, bundle $(AB12)$ is followed by $(AB21)$ for a δ small, while $(AB12)$ is followed by the product combination $(A2B2)$ for a δ high. In the same way, on the left half of the circle, the bundle $(AB21)$ is followed by $(AB12)$ for a δ small and straight followed by $(A1B1)$ for a δ high. Equilibrium prices and profits are computed in appendix 3. In

[13]See appendix 7.1.

the first case - for a low range of δ - they are given by:

$$p_A^* = c_A + \frac{1}{4}t_A - \frac{1}{6}t_B, \tag{10}$$
$$p_B^* = c_B + \frac{1}{12}t_B,$$
$$p_{AB}^{ij*} = c_A + c_B + \frac{1}{4}(t_A - t_B),$$
$$\Pi_{A_iB_j}^{2*} = \frac{1}{8}t_A - \frac{7}{72}t_B.$$

In the second case - for a high range of δ - equilibrium prices and profits are given by:

$$p_A^* = c_A + \frac{1}{4}t_A + (\frac{1}{2} - \delta)\frac{t_At_B}{3(t_A + t_B)}, \tag{11}$$
$$p_B^* = c_B + \frac{1}{4}t_B + (\frac{1}{2} - \delta)\frac{t_At_B}{3(t_A + t_B)},$$
$$p_{AB}^{ij*} = c_A + c_B + \frac{1}{4}(t_A + t_B),$$
$$\Pi_{A_iB_j}^{2*} = \frac{1}{8}(t_A + t_B) - (1 - 4(\delta^2 - \delta))\frac{t_At_B}{18(t_A + t_B)}.$$

Thus, we just have to calculate for which value of δ the demand configuration is changing. If both firms set equilibrium prices (10), then there exists a δ threshold from which bundle $(AB12)$ is no longer followed by $(AB21)$ but by $(A2B2)$. Calculating this threshold leads to $\delta = \delta_1^{HT} = \frac{1}{6} - \frac{tb}{6ta}$. On the other hand, if firms set equilibrium prices (11), then we find the threshold δ_2^{HT} beyond which the second demand configuration arises. This threshold is given by $\delta = \delta_2^{HT} = \frac{ta-tb}{5ta+tb}$. For more details, refer to appendix 7.1. For δ between $\frac{1}{6} - \frac{tb}{6ta}$ and $\frac{ta-tb}{5ta+tb}$, firms set prices in such a way that the last consumer purchasing $(AB12)$ is indifferent between $(AB12)$, $(AB21)$ and $(A2B2)$. Therefore, the first demand configuration still holds. The determination of the equilibrium prices in this region is similar to the one in a standard Hotelling model when we move from local monopoly to competition (Economides, 1984, Gabszewicz and Thisse, 1986). For $\frac{1}{6} - \frac{tb}{6ta} < \delta \leq \frac{ta-tb}{5ta+tb}$, equilibrium prices and profits are given by:

$$p_A^* = c_A + \frac{1}{4}t_A + \frac{t_At_B}{2(t_A - t_B)^2}(3(t_B - t_A) + 2\delta(8t_A + t_B)), \tag{12}$$
$$p_B^* = c_B + \frac{1}{4}t_B + \frac{t_At_B}{2(t_A - t_B)^2}(3(t_B - t_A) + 2\delta(8t_A + t_B)),$$
$$p_{AB}^{ij*} = c_A + c_B + \frac{1}{4}(t_A + t_B) + \frac{t_At_B}{2(t_A - t_B)^2}(6(t_B - t_A) + 6\delta(5t_A + t_B),$$
$$\Pi_{A_iB_j}^{2*} = \frac{1}{8}(t_A + t_B) + \frac{t_At_B}{2(t_A - t_B)^2}(3(t_B - t_A) + 2\delta(8t_A + t_B) - 4\delta^2t_A).$$

In comparison with the case of homogeneous mergers, intuitions about profit levels are the same but reversed relative to the correlation. In the heterogeneous merger case, bundles are composed of goods with opposite locations. Subsequently and contrary to the homogeneous merger case, the bundle competition effect appears for low correlation values.

In the same way, the sorting effect is also reversed relative to the correlation. Indeed, by sorting consumers, firms can make more profits than in the independent pricing case when the correlation is weak. Furthermore, when the correlation is high ($\delta < \delta_2^{HT}$), the competition effect between bundles appears and dominates the sorting effect. This is due to the opposite locations of merged firms on each market. We find a prisoner dilemma but now for high correlation values. Now that we studied competition outcomes corresponding to merger waves, the next section computes and analyzes the SPNE of the game.

4 Equilibrium of the game

In this section, we solve the whole game. Next, we analyze the welfare and competition policy implications.

4.1 Equilibrium computation

First, we deduce from lemma 1 and 3 the following proposition :

Proposition 2 *In equilibrium, a merger wave occurs and firms choose a mixed bundling strategy.*

Proof. In the homogeneous case and in the heterogeneous one, merger waves appear and the merged firms follow mixed bundling strategy (see section 3.1 and 3.2). The equilibrium of the whole game is therefore either a homogeneous merger wave or a heterogeneous merger one. ■

Thus, we are interested in outcomes which trigger a merger wave. As there can be only one type of merger at the same time, that is two homogeneous or two heterogeneous mergers, we compare the profits associated with these outcomes according to δ. As the game is symmetric, it is sufficient to compare the profit of firm A_1B_1 further to a homogeneous merger wave and the profit of firm A_1B_2 further to a heterogeneous one. In order to rank these equilibria profits according to δ, one must order the thresholds for which profit functions are modified. These thresholds are given by δ_1^{HM}, δ_2^{HM}, δ_1^{HT} and δ_2^{HT}. We already know that $\delta_1^{HT} < \delta_2^{HT}$ and $\delta_1^{HM} < \delta_2^{HM}$. Moreover, $\delta_2^{HT} = \frac{t_A - t_B}{5t_A + t_B} < \delta_1^{HM} = \frac{3}{2}(\frac{t_A + t_B}{5t_A + t_B})$. We deduce the following ranking: $\delta_1^{HT} < \delta_2^{HT} < \delta_1^{HM} < \delta_2^{HM}$. There is a turnover between the two types of merger according to the correlation of consumers reservation values for the two goods. This turnover occurs around three thresholds, *i.e.* δ_1^*, δ_2^* and δ_3^*. The profits comparison according to δ allows to compute the equilibrium and the associated profits. The following proposition[14] presents this equilibrium and figure 7 below illustrates the turnover between homogeneous and heterogeneous merger profits:

Proposition 3 *In equilibrium, firms make always more profits than in the independent pricing case. Moreover, firms choose to merge either in a homogeneous way, or in a heterogeneous one. This depends on the reservation prices correlation. They merge in a homogeneous way for* $0 \leq \delta \leq \delta_1^*$ *and* $\delta_2^* \leq \delta < \delta_3^*$ *and they merge in a heterogeneous one for* $\delta_1^* < \delta < \delta_2^*$ *and for* $\delta_3^* \leq \delta \leq \frac{1}{2}$.

[14] The proof is given in appendix 7.2.

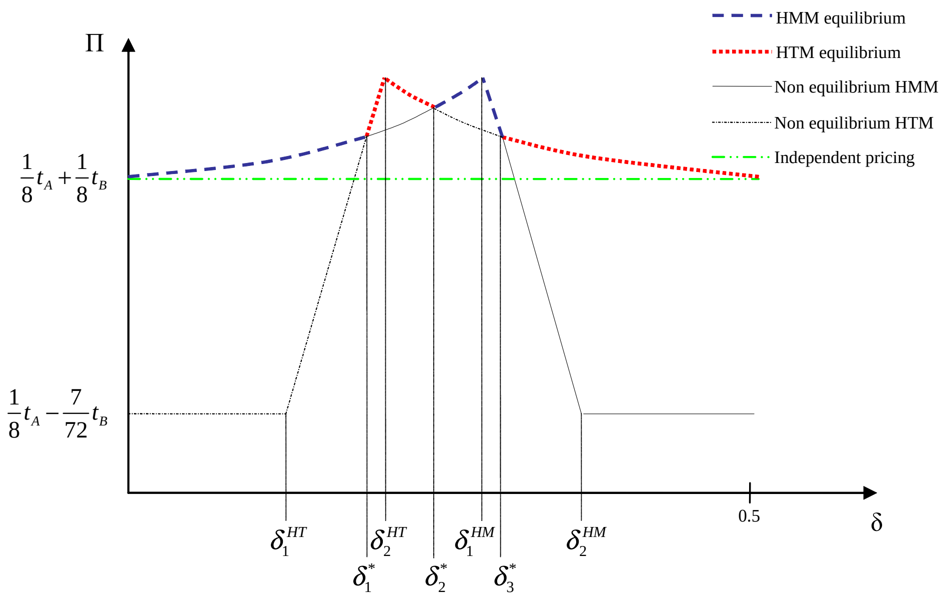

Figure 7: equilibrium profits

The intuitions behind the homogeneous merger wave and the heterogeneous one have already been explained. There is a trade off between a sorting effect due to bundling which is positive for the firms' point of view and a competition effect passing through product bundles ("business-stealing effect", Reisinger 2006). In the homogeneous merger case, this competition effect exists only if the correlation is weak. In the heterogeneous merger case, this competition effect exists only if the correlation is strong. Thus, when the correlation is sufficiently high ($\delta \leq \delta_1^*$), firms avoid this competition effect by merging in a homogeneous way. Conversely, if the correlation is sufficiently weak ($\delta \geq \delta_3^*$), firms avoid this competition effect by merging in a heterogeneous way. The sorting effect is maximum for a weaker correlation with heterogeneous mergers than with homogeneous ones. Therefore, firms choose to alternate between these two types of mergers[15] when the correlation is intermediate ($\delta_1^* < \delta < \delta_3^*$). This alternation is due to the following fact: according to the type of merger, the maximum profit values occur at different levels of correlation. These maximum values correspond to a strong sorting effect without triggering the business-stealing effect. Indeed, the sorting effect is even stronger than product bundles as the two firms are similar from consumers' point of view. This lack of differentiation between bundles finally creates this new competition effect.

Concerning the heterogeneous merger, and therefore the bundles of goods with opposite locations, the maximum value of profit reaches at $\delta_2^{HT} \leq \frac{1}{4}$. Concerning the heterogeneous merger, and therefore the bundles of goods with the same location, the maximum value of profit reaches at $\delta_1^{HM} \geq \frac{1}{4}$. In both cases, the different opportunities of merger allow firms to benefit better from the sorting effect by positionning their sales on the two

[15] To better illustrate these merger types, we give an example in telecommunications. The homogeneous merger could be a merger between the firm A_1 supplying mobile phone subscriptions and the firm B_2 providing internet accesses. The firm A_1 has the reputation to provide a vaster phone network than A_2 and the firm B_2 has the reputation to supply a more performant hotline than B_1. In the same way, the heterogeneous merger could be a merger between the firm A_2 and the firm B_1. The firm A_2 has a better reputation than A_1 regarding the mobile phone selection and the firm B_1 have a better internet network than B_2.

markets according to the correlation. Because of these merger opportunities, firms avoid the competition effect between bundles. At the equilibrium, it exists only for the two following ranges of parameters corresponding to "touching" equilibria: $\delta_1^* < \delta < \delta_2^{HT}$ and $\delta_1^{HM} < \delta < \delta_3^*$. Moreover, these ranges are very restricted since, respectively, they correspond at the most to $\frac{1}{5} - \frac{16}{10000} \approx \frac{36-3\sqrt{114}}{20} < \delta < \frac{1}{5}$ and $\frac{3}{10} < \delta < \frac{3\sqrt{114}-26}{20} \approx \frac{3}{10} + \frac{16}{10000}$. Such is the case when $t_B \longrightarrow 0$. Indeed, transportation costs vary the intensity of the two effects and the thresholds defining the different types of equilibria. Intuitively, the weaker the transportation cost t_B, the weaker the sorting and the competition effects. The two maximum profit values tend to get closer. Therefore, the minimum and maximum ranges of correlation become larger. To an extreme degree, when $t_B \longrightarrow 0$, bundling has no effect and the price of the good B is equal to its marginal cost of production. We find the Bertrand paradox since there is no horizontal differentiation of the good B and the sorting effect does not exist any more. Conversely, when $t_B \longrightarrow t_A$, the effects are intensified and for the two merger cases, the sorting effect is stronger and the competition effect between bundles (business stealing effect) takes place less easily. Indeed, the trade off between the low price of the bundle and the additional distance to cover for the good B tends to favor the separate goods consumption. Figure 8 below illustrates these two limit cases:

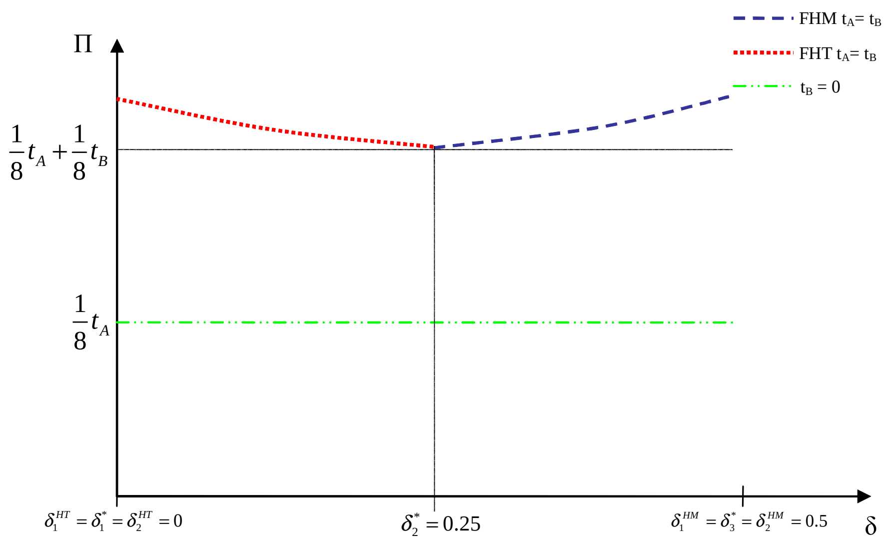

Figure 8: limit cases

The possibility for the four firms to merge with firms either at the same, or at the opposite location, eliminates the prisoner' dilemma already underlined in the two merger types. Indeed, as firms can choose their merger partner in relation to the correlation of reservation prices, firms can benefit better from the sorting effect without being affected by the competiton effect. In order to evaluate the scope of this study, we will focus on welfare implications in the following section.

4.2 Welfare analysis

First, we focus on social welfare. As a benchmark, we calculate the maximum welfare. So, the welfare is maximized when transportation costs are minimized. Indeed, price levels do not affect the social welfare because the volume of consumption is unchanged in this model. Maximum welfare is achieved when consumers who are located at x_k with $0 \leq x_k \leq \frac{1}{4}$ and $\frac{3}{4} \leq x_k \leq 1$ for $k = A$ (respectively $k = B$) buy good A from firm A_1 (respectively good B from firm B_1) and when consumers who are located at $\frac{1}{4} \leq x_k \leq \frac{3}{4}$ for $k = A$ (respectively $k = B$) buy good A from firm A_2 (respectively good B from firm B_2). This situation corresponds to the independent pricing case since the prices are such as covered distances are minimum. We note W^{IP} social welfare when firms follow an independent pricing strategy:

$$W^{IP} = W^0 - \frac{1}{48}(t_A + t_B), \tag{13}$$

with $W^0 = K_A + K_B - c_A - c_B$.

In this case, consumers always buy the goods who are near their locations. Therefore, welfare is maximum. Now, we focus on social welfare in case of merger waves. We describe social welfare at the game equilibrium in the following proposition[16] and in figure 9:

Proposition 4 *In equilibrium, social welfare is always lower than in the independent pricing case or than without merger. However, the loss in social welfare due to bundling is nuanced in comparison with the results of Reisinger (2006). This is due to the opportunity given to firms to merge in different ways according to the correlation between the two markets.*

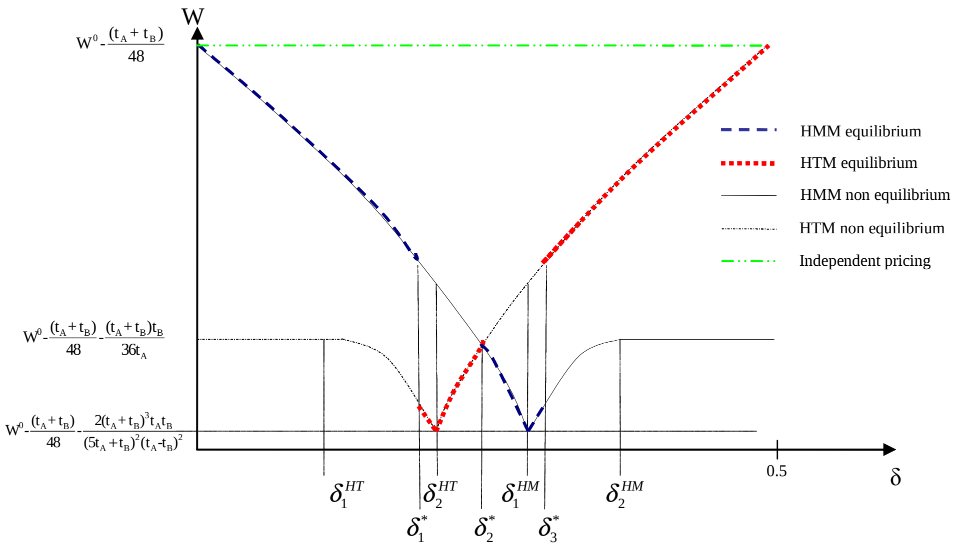

Figure 9: equilibrium welfare

We check the equilibrium social welfare is always lower than in cases where firms do not merge or sell their goods independently. The first recommendation is the following: competition authorities should prevent mergers allowing to bundle if one relies on our model assumptions. For instance, one might assume efficiency gains which could make merger advantageous from a social welfare point of view. Another way for authorities might be to forbid bundling. It is interesting to note that the possibility for firms to choose

[16]The proof of the proposition 4 is given in appendix 7.3.

a type of merger, homogeneous or heterogeneous, makes our results about welfare more balanced than those in Reisinger (2006). But, in accordance with Reisinger (2006), we note that bundling is harmful to social welfare. Indeed, the prisoner's dilemma disappears, which increases firms' profits when correlation is low. That increases social welfare. When the correlation is low ($\delta > \delta_3^*$), consumers choose packages with goods of far locations, thus there are lower transportation costs than in the model of Reisinger (2006). On the other hand, for the intermediate values of correlation, the possibility for firms to merge in homogeneous or heterogeneous way, can make profits higher but the transportation costs are considerably higher. That account for social welfare jumps in δ_1^* and δ_3^*. Finally, for high correlation values ($\delta < \delta_1^*$), we find the same equilibrium as Reisinger (2006) and the same level of social welfare. Moreover, we observe that the authorities generally prefer to take consumers' surplus into account rather than social welfare. Thus, we establish the following corollary[17] to proposition 4:

Corollary 1 *In equilibrium, the consumers' surplus is always lower than in the independent pricing case or without merger. The consumers' surplus analysis, as the social welfare one, leads to the following conclusion. In our framework, either bundling, or mergers leading to these strategies should be forbidden by authorities.*

Thus, we calculate the consumers' surplus. We note S^{IP} the consumers' surplus for the independent pricing case, corresponding to the maximum surplus case. It is easily given by social welfare minus twice merged firm profit in the independent pricing strategy:

$$S^{IP} = W^0 - \frac{13}{48}(t_A + t_B), \tag{14}$$

with $W^0 = K_A + K_B - c_A - c_B$.

We can conclude that the analysis of consumers' surplus gives the same results as the social welfare analysis. The firms' gain by bundling is insignificant in comparison with the effect due to bundling on social welfare. The graphical representation of equilibrium consumers' surplus is the same as the social welfare with a lower level and the slopes more pronounced. The analysis of consumers' surplus as social welfare carries on the same conclusion. Now, we present some concluding remarks.

5 Conclusion

Our paper studies bundling effects on mergers. More precisely, we show that bundling strategies can create incentives to merge. Indeed, competition effects of a merger involving firms from two independent markets are non-existent. This is no longer right if these firms can bundle.

First, we show there is always an incentive to follow a bundling strategy once a merger has been achieved. When bundling is not possible, we find there is no incentive to merge. From these two results, we deduce that bundling strategy generates not only a merger incentive but also a merger wave. Intuitively, this incentive comes from the sorting effect due to the bundling strategy. However, a competition effect, which is negative on firms' profits, is also generated by the bundling strategy. We find that, in order to take better

[17] The proof of corollary 1 is given in appendix 7.4.

advantage of the sorting effect and to avoid the competition effect, firms choose between two merger types. This trade-off is relative to the correlation of reservation prices for the two goods. A firm can merge with a firm at the same location on the other circle. A firm can also merge with another firm at an opposite location on the other circle. Respectively, we talk about homogeneous and heterogeneous mergers. These merger opportunities remove the prisoner's dilemma emphasized by Reisinger (2006). Indeed, this dilemma is created by the dominance of the competition effect.

The merger incentives provoked by bundling and analyzed in this theoretical paper reflect an empirical reality. We can provide merger cases which appear in part because of bundling strategies created by mergers. In 2004, the telecommunications company France Télécom bought out its past subsidiary Wanadoo. In the same way, in 2005, the german firm Deutsch Telekom took over T-Online. This same year in the United States, the telecommunications firm SBC bought AT&T. It is the same for Verizon that took over MCI. In each case, mergers allow involved firms to provide packages corresponding to double play, triple play, or quadruple play bundling strategies. These examples can be analyzed through the homogeneous and heterogeneous mergers concepts. This depends on merged firms' characteristics with regard to each good sold in packages.

Concerning competition policy, our model has implications. We show that bundling strategies have a negative effect on social welfare. As bundling is made possible after a merger process, the appropriate politics should be either to forbid mergers, or to forbid bundling. Competition authorities pay better attention to consumers' surplus than social welfare. Therefore, we also analyze the consumers' surplus in the case of independent pricing and in the case of bundling. However, this analysis gives no other interpretation. Finally, we show that the negative effect of bundling on social welfare is weaker than in Reisinger (2006).

Finally, we must note that our analysis framework dissociates pure effects of bundling and effects of anti-competitive mergers. A direction for future research on this topic could be the combination of these pure effects with other ones. For instance, efficiency gains can be introduced in the merger process. These effects might be altered or even reverse the results.

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7 Appendices

7.1 Heterogeneous mergers

7.1.1 Proof of the lemma 3

In the case of an heterogeneous merger wave, the firms A_i and B_j are merged $\forall i \neq j$ with $i, j = 1, 2$. Let us analyze if there is an incentive for the merged firm A_1B_2 to introduce a bundle. First, we consider the case where both firms do not bundle. Since the equilibrium is symmetric, both firms charge the same independent prices p_A^{IP} and p_B^{IP}, and earn profits of $\Pi^*_{A_iB_j} = \frac{1}{2}(p_A^{IP} - c_A + p_B^{IP} - c_B)$ $\forall i \neq j$ with $i, j = 1, 2$. Now, if firm A_1B_2 introduces a bundle, that means selling both goods together at a price $p_{AB}^{12} < p_A^1 + p_B^2$. We analyze the case where $p_{AB}^{12} = p_A^{IP} + p_B^{IP}$, but where $p_A^1 = p_A^{IP} + \varepsilon_1$ and $p_B^2 = p_B^{IP} + \varepsilon_1$, with $\varepsilon_1 > 0$ but small. So, the firm A_1B_2 increases its profits raising its independent prices by ε_1 and sets the bundle price equal to the sum of the independent prices.

We have to distinguish between two cases, either if δ is "near" $\frac{1}{2}$ or not, because this changes the demand structure on the circles. First look at the case where δ is not near $\frac{1}{2}$. If firms do not bundle there are four demand regions on the circles, namely $(A1B1)$, $(A1B2)$, $(A2B2)$, and $(A2B1)$. The frontiers between these regions (or the marginal consumers) are given by $\frac{1}{4} - \delta$ for the frontier between $(A1B1)$ and $(A1B2)$, by $\frac{1}{4}$ for $(A1B2)$ and $(A2B2)$, by $\frac{3}{4}$ for the frontier between $(A2B2)$ and $(A1B2)$ and finally by $\frac{3}{4} - \delta$ for $(A1B2)$ and $(A1B1)$.

If the firm A_1B_2 introduces the bundle $(AB12)$, the frontiers are changed to $\frac{1}{4} - \delta - \frac{\varepsilon_1}{t_B}$ for the frontier between $(A1B1)$ and $(AB12)$, to $\frac{1}{4} + \frac{\varepsilon_1}{t_A}$ for $(AB12)$ and $(A2B2)$, to $\frac{3}{4} - \frac{\varepsilon_1}{t_A}$ for the frontier between $(A2B2)$ and $(AB12)$ and finally to $\frac{3}{4} - \delta - \frac{\varepsilon_1}{t_B}$ for $(AB12)$ and $(A1B1)$. The new profit function of firm A_1B_2 is given by:

$$\begin{aligned} \Pi^{**}_{A_1B_2} &= (p_A^1 + p_B^2 - c_A - c_B)\left(2\varepsilon_1(\frac{1}{t_A} + \frac{1}{t_B})\right) + (p_A^1 - c_A + \varepsilon_1)\left(\frac{1}{2} - 2\varepsilon_1\frac{1}{t_B}\right) \qquad (15) \\ &+(p_B^2 - c_B + \varepsilon_1)\left(\frac{1}{2} - 2\varepsilon_1\frac{1}{t_A}\right), \end{aligned}$$

or

$$\Pi^{**}_{A_1B_2} = \Pi^*_{A_1B_2} + 2(p_A^1 - c_A)\frac{\varepsilon_1}{t_A} + 2(p_B^2 - c_B)\frac{\varepsilon_1}{t_B} + \varepsilon_1 - 2\frac{(\varepsilon_1)^2}{t_A} - 2\frac{(\varepsilon_1)^2}{t_B}$$

This profit is always higher than the previous profit $\Pi^*_{A_1B_2}$ as long as $\delta > 0$ because ε_1 can made arbitrary small and so $(\varepsilon_1)^2$ tends faster towards 0 than ε_1. We made the proof than the merged firm A_1B_2 has an incentive to introduce its bundle. Let us focus on firm A_2B_1 to introduce its bundle if the firm A_1B_2 is already bundling. The profit of firm A_2B_1 if firm A_1B_2 bundles while firm A_2B_1 not is given by:

$$\Pi^*_{A_2B_1} = (p_A^2 - c_A)\left(\frac{1}{2} - 2\varepsilon_1(\frac{1}{t_A})\right) + (p_B^1 - c_B)\left(\frac{1}{2} - 2\varepsilon_1(\frac{1}{t_B})\right)$$

If the firm A_2B_1 chooses to bundle $(AB21)$ and set $p_{AB}^{21} = p_A^{IP} + p_B^{IP}$, and sells their goods independently at the price $p_A^2 = p_A^{IP} + \varepsilon_2$ and $p_B^1 = p_B^{IP} + \varepsilon_2$, with $\varepsilon_2 > 0$ but small, the frontiers are given by $\frac{1}{4} - \delta - \frac{(\varepsilon_1 + \varepsilon_2)}{t_B}$for the frontier between $(A1B1)$ and $(AB12)$, by $\frac{1}{t_A - t_B}(\frac{1}{4}t_B - t_B\delta - \frac{1}{4}t_A)$ for $(AB12)$ and $(AB21)$, by $\frac{1}{4} - \delta + \frac{(\varepsilon_1 + \varepsilon_2)}{t_B}$for the frontier between

$(AB21)$ and $(A2B2)$, by $\frac{3}{4}-\delta-\frac{(\varepsilon_1+\varepsilon_2)}{tB}$ for $(A2B2)$ and $(AB21)$, by $\frac{1}{t_A-t_B}(-\frac{3}{4}t_A+\frac{3}{4}t_B-t_B\delta)$ for the frontier between $(AB21)$ and $(AB12)$ and finally by $\frac{3}{4}-\delta+\frac{(\varepsilon_1+\varepsilon_2)}{t_B}$ for $(AB12)$ and $(A1B1)$. The new profit function of firm A_2B_1, when it proposes bundles, is then:

$$
\begin{aligned}
\Pi^{**}_{A_2B_1} &= (p_A^2+p_B^1-c_A-c_B)\left(2(\varepsilon_1+\varepsilon_2)(\frac{1}{t_B})-\frac{1}{2}+\frac{1}{t_A-t_B}(\frac{1}{2}t_B-\frac{1}{2}t_A\right) \\
&\quad +(p_A^2-c_A+\varepsilon_2)\left(\frac{1}{2}-2(\varepsilon_1+\varepsilon_2)(\frac{1}{t_B})\right)+(p_B^1-c_B+\varepsilon_2)\left(\frac{1}{2}-2(\varepsilon_1+\varepsilon_2)(\frac{1}{t_B})\right) \\
&= \Pi^*_{A_2B_1}+\varepsilon_2-4\left(\frac{(\varepsilon_2)^2+\varepsilon_1\varepsilon_2}{t_B}\right).
\end{aligned}
\tag{16}
$$

Thus, for ε_1 and ε_2 small, bundling is profitable if $\delta>0$ since $(\varepsilon_2)^2$ and $(\varepsilon_1\varepsilon_2)$ tend faster towards 0 than ε_2. We shown that bundling is always a profitable strategy for δ small.

Now let us turn the case where δ is near $\frac{1}{2}$. First, we analyze the incentive of firm A_1B_2 to introduce its bundle while the other firm practises independent pricing. If firm A_1B_2 does not bundle, product combinations are: $(A1B2)$, $(A2B2)$, $(A2B1)$ and $(A1B1)$. The frontiers are given by $\frac{1}{4}$ for the frontier between $(A1B2)$ and $(A2B2)$, by $\frac{3}{4}-\delta$ for $(A2B2)$ and $(A2B1)$, by $\frac{3}{4}$ for the frontier between $(A2B1)$ and $(A1B1)$and finally by $\frac{5}{4}-\delta$ for $(A1B1)$ and $(A1B2)$.

If merged firm A_1B_2 bundles, then the frontiers are given by $\frac{1}{4}+\frac{\varepsilon_1}{t_A}$ for the frontier between $(AB12)$ and $(A2B2)$, by $\frac{3}{4}-\delta-\frac{\varepsilon_1}{t_B}$ for $(A2B2)$ and $(A2B1)$, by $\frac{3}{4}+\frac{\varepsilon_1}{t_A}$ for the frontier between $(A2B1)$ and $(A1B1)$ and finally by $\frac{5}{4}-\delta-\frac{\varepsilon_1}{t_B}$ for $(A1B1)$ and $(AB12)$. The profit of firm A_1B_2 if it bundles is:

$$
\begin{aligned}
\Pi^{**}_{A_1B_2} &= (p_A^1+p_B^2-c_A-c_B)\left(\varepsilon_1(\frac{1}{t_A}+\frac{1}{t_B})+\delta\right) \\
&\quad +(p_A^1-c_A+\varepsilon_1)\left(\frac{1}{2}-\varepsilon_1(\frac{1}{t_A}+\frac{1}{t_B})-\delta\right) \\
&\quad +(p_B^2-c_B+\varepsilon_1)\left(\frac{1}{2}-\varepsilon_1(\frac{1}{t_A}+\frac{1}{t_B})-\delta\right),
\end{aligned}
\tag{17}
$$

or

$$
\begin{aligned}
\Pi^{**}_{A_1B_2} &= (p_A^1-c_A+p_B^2-c_B)\left(\frac{1}{2}-\varepsilon_1(\frac{1}{t_A}+\frac{1}{t_B})-\delta\right) \\
&= \Pi^*_{A_1B_2}+\varepsilon_1(1-2\delta)-2(\varepsilon_1)^2\left(\frac{1}{t_A}+\frac{1}{t_B}\right).
\end{aligned}
\tag{18}
$$

$\Pi^{**}_{A_1B_2}$ is always higher than $\Pi^*_{A_1B_2}$ if $\delta>0$ since $(\varepsilon_1)^2$ tends faster towards 0 than ε_1. Therefore, A_1B_2 has an incentive to bundle. Now, let us analyze the profit of the firm A_2B_1 if the firm A_1B_2 is already bundling. If firm A_2B_1 chooses not to bundle, its profit is:

$$
\begin{aligned}
\Pi^*_{A_2B_1} &= (p_A^2+p_B^1-c_A-c_B)\left(\frac{1}{2}+\delta+\varepsilon_1(\frac{1}{t_A}+\frac{1}{t_B})\right) \\
&\quad +(p_A^2-c_A)\left(-\varepsilon_1(\frac{1}{t_A}+\frac{1}{t_B})-\delta\right)+(p_B^1-c_B)\left(-\varepsilon_1(\frac{1}{t_A}+\frac{1}{t_B})-\delta\right) \\
&= \frac{1}{2}(p_A^2+p_B^1-c_A-c_B).
\end{aligned}
\tag{19}
$$

If firm A_2B_1 also bundles, it sets prices $p^{21}_{AB} = p^{IP}_A + p^{IP}_B$, sells its goods separately at $p^2_A = p^{IP}_A + \varepsilon_2$ and $p^1_B = p^{IP}_B + \varepsilon_2$, with $\varepsilon_2 > 0$ but small. Now, the frontiers are given by $\frac{1}{4} + \frac{(\varepsilon_1+\varepsilon_2)}{t_A}$ for the frontier between $(AB12)$ and $(A2B2)$, by $\frac{3}{4} - \delta - \frac{(\varepsilon_1+\varepsilon_2)}{t_B}$ for $(A2B2)$ and $(AB21)$, by $\frac{3}{4} + \frac{(\varepsilon_1+\varepsilon_2)}{t_A}$ for the frontier between $(AB21)$ and $(A1B1)$ and finally by $\frac{5}{4} - \delta - \frac{(\varepsilon_1+\varepsilon_2)}{t_B}$ for $(A1B1)$ and $(AB12)$. Profit of firm A_2B_1 if both firms bundle is then:

$$\begin{aligned}\Pi^{**}_{A_2B_1} &= (p^2_A + p^1_B - c_A - c_B)\left(\frac{1}{2} + \varepsilon_1(\frac{1}{t_A} + \frac{1}{t_B}) + \varepsilon_2(\frac{1}{t_A} + \frac{1}{t_B}) + \delta\right) \\ &\quad + (p^2_A - c_A + \varepsilon_1)\left(-\varepsilon_1(\frac{1}{t_A} + \frac{1}{t_B}) - \varepsilon_2(\frac{1}{t_A} + \frac{1}{t_B}) - \delta\right) \\ &\quad + (p^1_B - c_B + \varepsilon_1)\left(-\varepsilon_1(\frac{1}{t_A} + \frac{1}{t_B}) - \varepsilon_2(\frac{1}{t_A} + \frac{1}{t_B}) - \delta\right),\end{aligned} \tag{20}$$

or

$$\Pi^{**}_{A_2B_1} = \Pi^{*}_{A_2B_1} + \varepsilon_2(1-2\delta) - (2\varepsilon_1\varepsilon_2)\left(\frac{1}{t_A} + \frac{1}{t_B}\right) - (2\varepsilon_2)^2\left(\frac{1}{t_A} + \frac{1}{t_B}\right). \tag{21}$$

If ε_1 and ε_2 are small, then $\Pi^{**}_{A_2B_1} > \Pi^{*}_{A_2B_1}$. Thus, firm A_2B_1 also has an incentive to bundle.

7.1.2 Proof of the consumption combinations:

In order to study the different equilibria of heterogeneous merger waves, we have to establish several claims concerning consumption combinations:

Claim 1 *There cannot exist direct rivalry between product combination* $(A1B1)$ *and* $(A2B2)$.

Proof. The method of proof is the same than for claim 1 of Reisinger (2006) ■

Claim 2 .
($\star$) *Take* x_A *and* x'_A *with* $0 \leq x_A, x'_A \leq \frac{1}{2}$ *and* $x'_A < x_A$.
If $(AB12)$ *is optimal at* x_A*, then* $(AB21)$ *can never be optimal at* x'_A.
($\star\star$) *Take* x_A *and* x'_A *with* $\frac{1}{2} \leq x_A, x'_A \leq 1$ *and* $x'_A < x_A$.
If $(AB21)$ *is optimal at* x_A*, then* $(AB12)$ *can never be optimal at* x'_A.

Proof. The method of proof is the same than for claim 2 of Reisinger (2006) ■

Claim 3 .
($\star$) *Take* x_A *and* x'_A *with* $0 \leq x_A, x'_A \leq \frac{1}{2}$ *and* $x'_A < x_A$.
If $(A1B1)$ *is optimal at* x_A*, then* $(A2B2)$ *can never be optimal at* x'_A.
($\star\star$) *Take* x_A *and* x'_A *with* $\frac{1}{2} \leq x_A, x'_A \leq 1$ *and* $x'_A < x_A$.
If $(A2B2)$ *is optimal at* x_A*, then* $(A1B1)$ *can never be optimal at* x'_A.

Proof. The method of proof is the same than for claim 3 of Reisinger (2006) ■

Claim 4 .
($\star$) *Take* x_A *and* x'_A *with* $0 \leq x_A, x'_A \leq \frac{1}{2}$ *and* $x'_A < x_A$.
If $(AB12)$ *is optimal at* x_A*, then* $(A2B2)$ *can never be optimal at* x'_A.
($\star\star$) *Take* x_A *and* x'_A *with* $\frac{1}{2} \leq x_A, x'_A \leq 1$ *and* $x'_A < x_A$.
If $(A2B2)$ *is optimal at* x_A*, then* $(AB12)$ *can never be optimal at* x'_A.

Proof. The method of proof is the same than for claim 4 of Reisinger (2006) ■

Claim 5 .
($\star$) *Take* x_A *and* x'_A *with* $0 \leq x_A, x'_A \leq \frac{1}{2}$ *and* $x'_A < x_A$.
If $(A1B1)$ *is optimal at* x_A*, then* $(AB21)$ *can never be optimal at* x'_A.
($\star\star$) *Take* x_A *and* x'_A *with* $\frac{1}{2} \leq x_A, x'_A \leq 1$ *and* $x'_A < x_A$.
If $(AB21)$ *is optimal at* x_A*, then* $(A1B1)$ *can never be optimal at* x'_A.

Proof. The method of proof is the same than for claim 5 of Reisinger (2006) ■

7.1.3 Proof of the equilibrium: heterogeneous mergers and strong correlation

Assuming δ small, we consider a consumer at location $x_A = 0$. If we move clockwise on circle A, then the consumer who is indifferent between $(A1B1)$ and $(AB12)$ is located at:

$$x_A = \frac{1}{4} + \frac{p^{12}_{AB} - p^1_A - p^1_B}{t_B} - \delta. \tag{22}$$

The product combination which is bought to the right of $(AB12)$ is $(AB21)$. The marginal consumer between to buy the bundle $(AB12)$ and the bundle $(AB21)$ is located at:

$$x_A = \frac{1}{t_A - t_B}(p^{21}_{AB} - p^{12}_{AB} + \frac{1}{4}(t_A - t_B) + t_B\delta). \tag{23}$$

Moving further to the right the next combination which is bought is $(A2B2)$ and the marginal consumer is located at:

$$x_A = \frac{1}{4} + \frac{p^2_A + p^2_B - p^{21}_{AB}}{t_B} - \delta. \tag{24}$$

If we pass the point 1/2 and move upward on the left side of the circle, we get the same product structure as on the right side, because of symmetry, only with firm A_i and B_j reversed. The profit function of firm A_1B_2 is therefore:

$$\begin{aligned}
\Pi_{A_1B_2} \;=\; & (p^1_A - c_A)(\frac{1}{4} + \frac{p^{12}_{AB} - p^1_A - p^1_B}{t_B} - \delta + \frac{1}{4} + \delta - \frac{p^{12}_{AB} - p^1_A - p^1_B}{t_B}) \\
& +(p^{12}_{AB} - c_A - c_B)(\frac{p^{21}_{AB} - p^{12}_{AB} + \frac{1}{4}(t_A - t_B) + t_B\delta}{t_A - t_B} - \frac{1}{4} - \frac{p^1_A + p^1_B - p^{12}_{AB}}{t_B} + \delta \\
& + \frac{3}{4} + \frac{p^1_A + p^1_B - p^{12}_{AB}}{t_B} - \delta - \frac{p^{12}_{AB} - p^{21}_{AB} + \frac{3}{4}(t_A - t_B) + t_B\delta)}{t_A - t_B}) \\
& +(p^2_B - c_B)(\frac{3}{4} + \frac{p^{21}_{AB} - p^2_A - p^2_B}{t_B} - \delta - \frac{1}{4} - \frac{p^2_A + p^2_B - p^{21}_{AB}}{t_B} + \delta).
\end{aligned} \tag{25}$$

Because of symmetry we get a similar function for firm A_2B_1. Calculating prices and profits, for both firms, we get:

$$\begin{aligned}
p^*_A &= c_A + \frac{1}{4}t_A - \frac{1}{6}t_B, \\
p^*_B &= c_B + \frac{1}{12}t_B, \\
\forall i \neq j \qquad p^{ij*}_{AB} &= c_A + c_B + \frac{1}{4}(t_A - t_B), \\
\forall i \neq j \qquad \Pi^{z=2*}_{A_iBj} &= \frac{1}{8}t_A - \frac{7}{72}t_B.
\end{aligned} \tag{26}$$

with $i, j = 1, 2$. When the value of δ increases, there is a disappearance of bundle options. It exist a value of δ where the bundle $(AB12)$ is not following by the bundle $(AB21)$. At this threshold demand structure changes and there is only one option of bundle consumption.

7.1.4 Proof of the equilibrium: heterogeneous mergers and weak correlation

Assuming δ high, we consider a consumer at location $x_A = 0$. If we move clockwise on circle A, there is a consumer who is indifferent between $(AB12)$ and $(A2B2)$ located at:

$$x_A = \frac{1}{4} + \frac{p_A^2 + p_B^2 - p_{AB}^{12}}{t_A}. \tag{27}$$

If we move clockwise on circle A, the next product combination is $(AB21)$. So, The marginal consumer between $(A2B2)$ and $(AB21)$ is located at:

$$x_A = \frac{3}{4} + \frac{p_{AB}^{21} - p_A^2 - p_B^2}{t_B} - \delta. \tag{28}$$

If we pass the point $1/2$ and move upward on the left side of the circle A, we get the same product structure as on the right side, because of symmetry, only with firm A_i and B_j reversed. Therefore, the profit function of firm A_1B_2 is:

$$\begin{aligned}\Pi_{A_1B_2} = {} & (p_{AB}^{12} - c_A - c_B)(\frac{1}{4} + \frac{p_A^2 + p_B^2 - p_{AB}^{12}}{t_A} - \frac{1}{4} - \frac{p_{AB}^{12} - p_A^1 - p_B^1}{t_B} + \delta) \\ & + (p_A^1 - c_A)(\frac{5}{4} + \frac{p_{AB}^{12} - p_A^1 - p_B^1}{t_B} - \delta - \frac{3}{4} + \frac{p_A^1 + p_B^1 - p_{AB}^{21}}{t_A}) \\ & + (p_B^2 - c_B)(\frac{3}{4} + \frac{p_{AB}^{21} - p_A^2 - p_B^2}{t_B} - \delta - \frac{1}{4} - \frac{p_A^2 + p_B^2 - p_{AB}^{12}}{t_A}).\end{aligned} \tag{29}$$

Because of symmetry we get a similar function for firm A_2B_1. Calculating prices and profits, for both firms, we get:

$$\begin{aligned} & p_A^* = c_A + \frac{1}{4}t_A + (\frac{1}{2} - \delta)\frac{t_A t_B}{3(t_A + t_B)}, \\ & p_B^* = c_B + \frac{1}{4}t_B + (\frac{1}{2} - \delta)\frac{t_A t_B}{3(t_A + t_B)}, \\ & \forall i \neq j \qquad p_{AB}^{ij*} = c_A + c_B + \frac{1}{4}(t_A + t_B), \\ & \forall i \neq j \qquad \Pi_{A_iB_j}^{z=2*} = \frac{1}{8}(t_A + t_B) - (1 - 4(\delta^2 - \delta))\frac{t_A t_B}{18(t_A + t_B)}. \end{aligned} \tag{30}$$

7.1.5 Proof of the intermediate thresholds: heterogeneous mergers

For the profit function (26) arises, $(A1B1)$ must be followed by $(AB12)$ and not by $(AB21)$. The frontier between $(A1B1)$ and $(AB12)$ at the equilibrium prices is given by:

$$x_A = \frac{1}{12} - \delta. \tag{31}$$

The frontier between $(A1B1)$ and $(AB21)$ at the equilibrium prices is given by:

$$x_A = \frac{1}{4} + \frac{t_B \delta}{t_A - t_B}. \tag{32}$$

For the demand structure (*iii*) arises, then (31) must be smaller than (32). This gives the first threshold:

$$\delta_1^{HT} = \frac{1}{6} - \frac{t_B}{6t_A}. \tag{33}$$

For the profit profit function (29) to arise, the option consumption $(A1B1)$ must be followed by $(AB21)$ and not by $(AB12)$. Calculating in the same way as before by inserting the equilibrium prices corresponding to the profit function (29) in (31) and (32) gives that demand structure (*iv*) arises only if:

$$\delta > \delta_2^{HT} = \frac{ta - tb}{5ta + tb}. \tag{34}$$

7.1.6 Proof of the intermediate equilibrium: heterogeneous mergers

In the region such as $\frac{1}{6} - \frac{t_B}{6t_A} < \delta \leq \frac{ta-tb}{5ta+tb}$, firms set their prices in such a way that demand structure (*iv*) arises. Prices are determined in order to satisfy the following constraint:

$$\frac{1}{4} + \frac{p_A^2 + p_B^2 - p_{AB}^{12}}{t_A} \leq \frac{3}{4} + \frac{p_{AB}^{21} - p_A^2 - p_B^2}{t_B} - \delta \tag{35}$$

This means that $(A2B2)$ is followed by $(AB12)$ and not by $(AB21)$, and that thresholds δ_1^{HT} and δ_2^{HT} are given by (10) and (11) respectively. A linear interpolation between prices given by (10) for δ_1^{HT} and by (11) for δ_2^{HT} gives the following equilibrium prices and profits:

$$\begin{aligned}
p_A^* &= c_A + \frac{1}{4}t_A + \frac{t_A t_B}{2(t_A - t_B)^2}(3(t_B - t_A) + 2\delta(8t_A + t_B)), \qquad (36)\\
p_B^* &= c_B + \frac{1}{4}t_B + \frac{t_A t_B}{2(t_A - t_B)^2}(3(t_B - t_A) + 2\delta(8t_A + t_B)),\\
\forall i &\neq j \quad p_{AB}^{ij*} = c_A + c_B + \frac{1}{4}(t_A + t_B) + \frac{t_A t_B}{2(t_A - t_B)^2}(6(t_B - t_A) + 6\delta(5t_A + t_B),\\
\forall i &\neq j \quad \Pi_{A_iBj}^{z=2*} = \frac{1}{8}t_A + \frac{1}{8}t_B + \frac{t_A t_B}{2(t_A - t_B)^2}(3(t_B - t_A) + 2\delta(8t_A + t_B) - 4\delta^2 t_A).
\end{aligned}$$

We check these prices and profits are equilibrium ones for a heterogeneous merger wave.

7.2 Proof of the proposition 3

Profits corresponding to homogeneous and heterogeneous merger waves are equal for three values of δ parameter.

Indeed, for $\delta_1^* = \frac{3(24t_A^2+27t_At_B+3t_B^2-\sqrt{21t_B^4+246t_B^3t_A+981t_B^2t_A^2+1212t_A^3t_B+456t_A^4})}{4(10t_A^2+7t_At_B+t_B^2)}$, with $\delta_1^{HT} < \delta_1^* < \delta_1^{HT}$, the heterogeneous merger profit corresponding to the equilibrium (12) is equal to the homogeneous merger profit corresponding to the equilibrium (7). After δ_1^*, the equilibrium profit (12) becomes greater than the equilibrium profit (7).

Next, for $\delta_2^* = \frac{1}{4}$, with $\delta_2^{HT} < \delta_2^* < \delta_1^{HM}$, the heterogeneous merger profit corresponding to the equilibrium (11) is equal to the homogeneous merger profit corresponding to the equilibrium (7). After δ_2^*, the equilibrium profit (7) becomes greater than to the equilibrium profit (11).

Finally, for $\delta_3^* = \frac{-(52t_A^2+67t_At_B+7t_B^2-\sqrt{21t_B^4+246t_B^3t_A+981t_B^2t_A^2+1212t_A^3t_B+456t_A^4})}{4(10t_A^2+7t_At_B+t_B^2)}$, with $\delta_1^{HM} < \delta_3^* < \delta_2^{HM}$, the heterogeneous merger profit corresponding to the equilibrium (11) is equal to the homogeneous merger profit corresponding to the equilibrium (8). After δ_3^*, the equilibrium profit (11) becomes greater than the equilibrium profit (8).

It is easy to prove these different profit functions are monotonic for considered parameter values. Thus, profit differences increase or decrease as specified here. It is also easy to prove that there is only for these three values of δ, that are δ_1^*, δ_2^*, and δ_3^*, that profits of heterogeneous and homogeneous merger wavesbecome level. Moreover, for $t_A \geq t_B \geq 0$, the following ranking is still valid: $0 \leq \delta_1^{HT} \leq \delta_2^{HT} \leq \delta_2^* = \frac{1}{4} \leq \delta_1^{HM} \leq \delta_3^* \leq \delta_2^{HM} \leq \frac{1}{2}$. We can deduce the following lemma:

Lemma 4 *If $0 \leq \delta \leq \delta_1^*$, in equilibrium, firms choose to merge in a homogeneous way. Prices and profits are given by (7).*

If $\delta_1^* < \delta < \delta_2^*$, in equilibrium, firms choose to merge in a heterogeneous way. Prices and profits are given by (12) for $\delta_1^* < \delta < \delta_2^{HT}$ and by (11) for $\delta_2^{HT} \leq \delta < \delta_2^*$.

If $\delta_2^* \leq \delta < \delta_3^*$, in equilibrium, firms choose to merge in a homogeneous way. Prices and profits are given by (7) for $\delta_2^* \leq \delta < \delta_1^{HM}$ and by (8) for $\delta_1^{HM} < \delta < \delta_3^*$.

If $\delta_3^* \leq \delta \leq \frac{1}{2}$, in equilibrium, firms choose to merge in a heterogeneous way. Prices and profits are given by (11).

7.3 Proof of the social welfare

7.3.1 Proof of the social welfare: homogeneous mergers

See Reisinger (2006).

7.3.2 Proof of the social welfare: heterogeneous mergers

As in the previous appendix, we compute the welfare by integrating equilibrium prices in every frontiers of consumption combinations and by calculating total transportation costs. It is not necessary to compute the welfare for $\delta < \frac{1}{6} - \frac{t_B}{6t_A}$ since it is never the equilibrium welfare. Here, we compute the welfare when $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ and when $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$. If $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$, we get:

$$\begin{aligned} W_2^{HT} &= K_A + K_B - c_A - c_B \\ &-t_A \left(\int_0^{-\frac{1}{4}\frac{t_B-4t_B\delta-t_A}{t_A-t_B}} (x)^2 dx + \int_{-\frac{1}{4}\frac{t_B-4t_B\delta-t_A}{t_A-t_B}}^{\frac{1}{2}} (\frac{1}{2}-x)^2 dx \right) \\ &-t_A \left(\int_{\frac{1}{2}}^{-\frac{1}{4}\frac{3t_B-4t_B\delta-3t_A}{t_A-t_B}} (x-\frac{1}{2})^2 dx + \int_{-\frac{1}{4}\frac{3(t_B-t_A)-4t_B\delta}{t_A-t_B}}^{1} (1-x)^2 dx \right) \\ &-t_B \left(\int_0^{-\frac{1}{4}\frac{3(t_B-t_A)-4t_B\delta+8t_A\delta}{t_A-t_B}+\delta-1} (x)^2 dx + \int_{-\frac{1}{4}\frac{5(t_B-t_A)-4t_B\delta+8t_A\delta}{t_A-t_B}+\delta-1}^{\frac{1}{2}} (\frac{1}{2}-x)^2 dx \right) \\ &-t_B \left(\int_{\frac{1}{2}}^{-\frac{1}{4}\frac{3(t_B-t_A)-4t_B\delta+8t_A\delta}{t_A-t_B}+\delta} (x-\frac{1}{2})^2 dx + \int_{-\frac{1}{4}\frac{3(t_B-t_A)-4t_B\delta+8t_A\delta}{t_A-t_B}+\delta}^{1} (1-x)^2 dx \right). \end{aligned} \tag{37}$$

After some manipulations, we get:

$$\begin{aligned} W_2^{HT} &= K_A + K_B - c_A - c_B - \frac{1}{48}(t_A+t_B) \\ &\quad -\frac{t_A t_B}{(t_A-t_B)}(\delta^2-\delta) - \frac{1}{24}\frac{(t_A^2 - 3t_At_B + 2t_B^2)}{(t_A-t_B)} \\ &= W^{IP} - \frac{t_A t_B}{(t_A-t_B)}(\delta^2-\delta) - \frac{1}{24}\frac{(t_A^2 - 3t_At_B + 2t_B^2)}{(t_A-t_B)}. \end{aligned} \tag{38}$$

Finally, for $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$, the welfare is given by:

$$\begin{aligned} W_3^{HT} &= K_A + K_B - c_A - c_B \\ &-t_A \left(\int_0^{\frac{1}{12}\frac{3t_A+7t_B-8t_B\delta}{t_A+t_B}} (x)^2 dx + \int_{\frac{1}{12}\frac{3t_A+7t_B-8t_B\delta}{t_A+t_B}}^{\frac{1}{2}} (\frac{1}{2}-x)^2 dx \right) \\ &-t_A \left(\int_{\frac{1}{2}}^{\frac{1}{12}\frac{9t_A+13t_B-8t_B\delta}{t_A+t_B}} (x-\frac{1}{2})^2 dx + \int_{\frac{1}{12}\frac{9t_A+13t_B-8t_B\delta}{t_A+t_B}}^{1} (1-x)^2 dx \right) \\ &-t_B \left(\int_0^{-\frac{-11t_A+4t_A\delta-15t_B+12t_B\delta}{12(t_A+t_B)}+\delta-1} (x)^2 dx + \int_{-1\frac{-11t_A+4t_A\delta-15t_B+12t_B\delta}{12(t_A+t_B)}+\delta-1}^{\frac{1}{2}} (\frac{1}{2}-x)^2 dx \right) \\ &-t_B \left(\int_{\frac{1}{2}}^{-\frac{1}{12}\frac{-5t_A+4t_A\delta-9t_B+12t_B\delta}{t_A+t_B}+\delta} (x-\frac{1}{2})^2 dx + \int_{-\frac{1}{12}\frac{-5t_A+4t_A\delta-9t_B+12t_B\delta}{t_A+t_B}+\delta}^{1} (1-x)^2 dx \right). \end{aligned} \tag{39}$$

After some manipulations, we get :

$$\begin{aligned} W_3^{HT} &= K_A + K_B - c_A - c_B - \frac{1}{48}(t_A+t_B) \\ &-\frac{4}{9}\frac{(t_At_B^2\delta^2)}{(t_A+t_B)^2} + \frac{1}{144}\frac{(48t_A^2t_B+112t_At_B^2)\delta}{(t_A+t_B)^2} \\ &-\frac{1}{144}\frac{6t_A^3+33t_A^2t_B+40t_At_B^2-3t_B^3}{(t_A+t_B)^2} \\ &= W^{IP} \\ &-\frac{4}{9}\frac{(t_At_B^2\delta^2)}{(t_A+t_B)^2} + \frac{1}{144}\frac{(48t_A^2t_B+112t_At_B^2)\delta}{(t_A+t_B)^2} \\ &-\frac{1}{144}\frac{6t_A^3+33t_A^2t_B+40t_At_B^2-3t_B^3}{(t_A+t_B)^2}. \end{aligned} \tag{40}$$

7.3.3 Proof of the social welfare at game equilibrium with bundling

The appendices 7.3.1 and 7.3.2, as the lemma 4 allow to determine the social welfare at equilibrium game.

Lemma 5 *In equilibrium, the social welfare is given by:*

$W_1^{HM} = W^{IP} - \frac{4}{9}\delta^2\frac{t_At_B}{t_A+t_B}$ if $0 \le \delta \le \delta_1^*$,
$W_2^{HT} = W^{IP} - \frac{t_At_B}{(t_A-t_B)}(\delta^2-\delta) - \frac{1}{24}\frac{(t_A^2-3t_At_B+2t_B^2)}{(t_A-t_B)}$ if $\delta_1^* < \delta < \delta_2^{HT}$,
$W_3^{HT} = W^{IP} - \frac{4}{9}\frac{t_At_B^2\delta^2}{(t_A+t_B)^2} + \frac{\delta}{144}\frac{48t_A^2t_B+112t_At_B^2}{(t_A+t_B)^2} - \frac{1}{144}\frac{6t_A^3+33t_A^2t_B+40t_At_B^2-3t_B^3}{(t_A+t_B)^2}$ if $\delta_2^{HT} \le \delta < \delta_2^*$,
$W_1^{HM} = W^{IP} - \frac{4}{9}\delta^2\frac{t_At_B}{t_A+t_B}$ if $\delta_2^* \le \delta < \delta_1^{HM}$,

$W_2^{HM} = W^{IP} - (\frac{1}{4} + \delta^2 - \delta)\frac{(t_A+t_B)t_At_B}{(t_A-t_B)^2}$ if $\delta_1^{HM} < \delta < \delta_3^*$,
$W_3^{HT} = W^{IP} - \frac{4}{9}\frac{(t_At_B^2\delta^2)}{(t_A+t_B)^2} + \frac{1}{144}\frac{(48t_A^2t_B+112t_At_B^2)\delta}{(t_A+t_B)^2} - \frac{1}{144}\frac{6t_A^3+33t_A^2t_B+40t_At_B^2-3t_B^3}{(t_A+t_B)^2}$ if $\delta_3^* \leq \delta \leq \frac{1}{2}$.

7.4 Proof of the consumers' surplus

7.4.1 Proof of the consumers' surplus: homogeneous mergers

We just subtract two equilibrium profits from equilibrium welfare to find the consumers' surplus. For a homogeneous merger wave and for $\delta < \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$, the consumers' surplus is given by:

$$\begin{aligned} S_1^{HM} &= K_A + K_B - c_A - c_B - \frac{13}{48}(t_A+t_B) \\ &\quad -\frac{39}{72}(t_A+t_B) - \frac{8}{9}\delta^2\frac{t_At_B}{t_A+t_B} \\ &= S^{IP} - \frac{39}{72}(t_A+t_B) - \frac{8}{9}\delta^2\frac{t_At_B}{t_A+t_B}. \end{aligned} \tag{41}$$

For a homogeneous merger wave and for $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta < \frac{1}{3} + \frac{t_B}{6t_A}$, the consumers' surplus is given by:

$$\begin{aligned} S_1^{HM} &= K_A + K_B - c_A - c_B - \frac{13}{48}(t_A+t_B) \\ &\quad -\frac{2}{48}\frac{(13t_A^3+89t_Bt_A^2+89t_At_B^2+13t_B^3)}{(t_A-t_B)^2} - \frac{2}{48}\delta\frac{312t_Bt_A^2+72t_At_B^2}{(t_A-t_B)^2} \\ &\quad -\frac{2}{48}\delta^2\frac{24t_At_B^2-72t_Bt_A^2}{(t_A-t_B)^2} \\ &= S^{IP} \\ &\quad -\frac{2}{48}\frac{(13t_A^3+89t_Bt_A^2+89t_At_B^2+13t_B^3)}{(t_A-t_B)^2} - \frac{2}{48}\delta\frac{312t_Bt_A^2+72t_At_B^2}{(t_A-t_B)^2} \\ &\quad -\frac{2}{48}\delta^2\frac{24t_At_B^2-72t_Bt_A^2}{(t_A-t_B)^2}. \end{aligned} \tag{42}$$

7.4.2 Proof of the consumers' surplus: heterogeneous mergers

We just subtract two equilibrium profits from equilibrium welfare to find the consumers' surplus. For a heterogeneous merger wave and for $\frac{1}{6} - \frac{t_B}{6t_A} < \delta < \frac{ta-tb}{5ta+tb}$, the consumers' surplus is given by:

$$
\begin{aligned}
S_2^{HT} &= K_A + K_B - c_A - c_B - \frac{13}{48}(t_A + t_B) \\
&\quad -\frac{2}{48}\frac{(13t_A^3 - 85t_B t_A^2 + 59t_A t_B^2 + 13t_B^3)}{(t_A - t_B)^2} - \frac{2}{48}\delta\frac{384t_B t_A^2 + 48t_A t_B^2}{(t_A - t_B)^2} \\
&\quad -\frac{2}{48}\delta^2\frac{24t_A t_B^2 - 72t_B t_A^2}{(t_A - t_B)^2} \\
&= S^{IP} \\
&\quad -\frac{2}{48}\frac{(13t_A^3 - 85t_B t_A^2 + 59t_A t_B^2 + 13t_B^3)}{(t_A - t_B)^2} - \frac{2}{48}\delta\frac{384t_B t_A^2 + 48t_A t_B^2}{(t_A - t_B)^2} \\
&\quad -\frac{2}{48}\delta^2\frac{24t_A t_B^2 - 72t_B t_A^2}{(t_A - t_B)^2}.
\end{aligned} \tag{43}
$$

For a heterogeneous merger wave and for $\delta > \frac{ta-tb}{5ta+tb}$ the consumers' surplus is given by:

$$
\begin{aligned}
S_3^{HT} &= K_A + K_B - c_A - c_B - \frac{13}{48}(t_A + t_B) \\
&\quad -\frac{1}{72}\frac{(39t_A^2 + 94t_A t_B + 39t_B^2)}{(t_A + t_B)} - \frac{8}{9}(\delta + \delta^2)\frac{t_A t_B}{(t_A + t_B)} \\
&= S^{IP} \\
&\quad -\frac{1}{72}\frac{(39t_A^2 + 94t_A t_B + 39t_B^2)}{(t_A + t_B)} - \frac{8}{9}(\delta + \delta^2)\frac{t_A t_B}{(t_A + t_B)}.
\end{aligned} \tag{44}
$$

7.4.3 Proof of the consumers' surplus at equilibrium game with bundling

The appendices 7.4.1 and 7.4.2, as the lemma 4, allow to determine the consumers' surplus at the equilibrium.

Lemma 6 *In equilibrium, the consumers' surplus is given by:*

$S_1^{HM} = S^{IP} - \frac{39}{72}(t_A + t_B) - \frac{8}{9}\delta^2\frac{t_A t_B}{t_A + t_B}$ if $0 \leq \delta \leq \delta_1^*$,
$S_2^{HT} = S^{IP} - \frac{13t_A^3 - 85t_B t_A^2 + 59t_A t_B^2 + 13t_B^3}{24(t_A - t_B)^2} - \delta\frac{384t_B t_A^2 + 48t_A t_B^2}{24(t_A - t_B)^2} - \delta^2\frac{24t_A t_B^2 - 72t_B t_A^2}{24(t_A - t_B)^2}$ if $\delta_1^* < \delta < \delta_2^{HT}$,
$S_3^{HT} = S^{IP} - \frac{1}{72}\frac{(39t_A^2 + 94t_A t_B + 39t_B^2)}{(t_A + t_B)} - \frac{8}{9}(\delta + \delta^2)\frac{t_A t_B}{(t_A + t_B)}$ if $\delta_2^{HT} \leq \delta < \delta_2^*$,
$S_1^{HM} = S^{IP} - \frac{39}{72}(t_A + t_B) - \frac{8}{9}\delta^2\frac{t_A t_B}{t_A + t_B}$ if $\delta_2^* \leq \delta < \delta_1^{HM}$,
$S_2^{HM} = S^{IP} - \frac{13t_A^3 + 89t_B t_A^2 + 89t_A t_B^2 + 13t_B^3}{24(t_A - t_B)^2} - \delta\frac{312t_B t_A^2 + 72t_A t_B^2}{24(t_A - t_B)^2} - \delta^2\frac{24t_A t_B^2 - 72t_B t_A^2}{24(t_A - t_B)^2}$ if $\delta_1^{HM} < \delta < \delta_3^*$,
$S_3^{HT} = S^{IP} - \frac{1}{72}\frac{39t_A^2 + 94t_A t_B + 39t_B^2}{(t_A + t_B)} - \frac{8}{9}(\delta + \delta^2)\frac{t_A t_B}{(t_A + t_B)}$ if $\delta_3^* \leq \delta \leq \frac{1}{2}$.