

# *Cahiers du LASER*

n°26-01-09

## **Cost-based access regulation and collusion in a differentiated duopoly**

*Edmond Baranes et Jean-Christophe Poudou*

Université Montpellier 1, LASER

### ***Laboratoire de Sciences Economiques de Richter***

UNIVERSITE DE MONTPELLIER I - Faculté des Sciences Economiques,  
Espace Richter - Avenue de la mer, CS 79606 - 34960 Montpellier Cedex, France

Tel : (33) 04 67 15 84 50 Fax : (33) 04 67 15 83 61

E-mail : ebaranes@univ-montp1.fr Web : <http://www.sceco.univ-montp1.fr/laser/>

# Accepted Manuscript

Cost-based access regulation and collusion in a differentiated duopoly

Edmond Baranes, Jean-Christophe Poudou

PII: S0165-1765(09)00374-7  
DOI: doi: [10.1016/j.econlet.2009.11.012](https://doi.org/10.1016/j.econlet.2009.11.012)  
Reference: ECOLET 4333

To appear in: *Economics Letters*

Received date: 16 July 2008  
Revised date: 17 November 2009  
Accepted date: 18 November 2009



Please cite this article as: Baranes, Edmond, Poudou, Jean-Christophe, Cost-based access regulation and collusion in a differentiated duopoly, *Economics Letters* (2009), doi: [10.1016/j.econlet.2009.11.012](https://doi.org/10.1016/j.econlet.2009.11.012)

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# Cost-based access regulation and collusion in a differentiated duopoly\*

Edmond Baranes<sup>†</sup> and Jean-Christophe Poudou<sup>‡</sup>

November 17, 2009

## Abstract

This paper revisits the conventional doctrine that "it is easier to collude among equals", applied in the context of vertically-related markets. In a differentiated duopoly model, we study how cost-based access price regulation may hinder the sustainability of tacit collusion.

JEL Classification: L10, L50

Keywords: Collusion, access, regulation, vertical structure.

## 1 Introduction

Symmetry among firms has been identified as a crucial factor influencing sustainability of tacit collusion in industrial settings. This has been mainly motivated by two arguments. First, symmetry makes easier to reach a common agreement based on prices. Second, symmetry makes easier the detection of deviations and the implementation of punishments. Therefore, effects of cost asymmetries have been largely considered in the literature on collusion (see for example Ivaldi *et al.* 2003). Traditional results show that low-cost firms in the industry have less incentives to collude and that cost asymmetry hinders collusion. This leads to the conventional doctrine that "it is easier to collude among equals".

Our paper examines this result in the case of vertically related industries in which firms supply differentiated products and for which access to an essential facility (e.g. a transportation network) is a key factor for competition to take

---

\*Thanks to an anonymous referee for thoughtful comments.

<sup>†</sup>Corresponding author: Edmond Baranes. Faculty of Economics, University of Montpellier, Espace Richter, av. de la Mer, CS 79606, 34960 Montpellier cedex 2, France. Tel.: +33 467 158 317, Fax: +33 467 158 361. Email address: edmond.baranes@univ-montp1.fr.

<sup>‡</sup>Faculty of Economics, University of Montpellier.

place. A large stream of literature emphasizes how product differentiation acts on sustainability of collusion in various competitive settings. Significant contributions are Deneckere (1983), Rothschild, (1992) and Collie (2006). Only a few papers examine the interplay between access pricing, regulation and collusion in downstream markets. Furthermore, an extended literature has been devoted to the analysis of regulatory issues in network industries, see Armstrong and Sappington (2007), for a survey. In particular, it is emphasized that to set the access charge at the cost of providing access, known as cost-based regulation<sup>1</sup>, is a common and practical regulatory policy. Hence in these industries, cost-based regulation leads to consider access charge as a cost symmetry parameter which can be controlled by national regulatory authorities. In such a context, it is interesting to study whether the conventional doctrine would be put back into question by cost-based access regulation.

We consider a differentiated duopoly model where an integrated firm (the incumbent) offers an access to an essential facility to a competitor. We use a standard infinitely repeated game, where both firms maximize the present discounted value of future profits. In this setting, we show that the conventional doctrine is not always at work when there are asymmetries in price-sensitivity of demand<sup>2</sup> in the industry. In particular, it does not apply when the competitor's demand is more sensible to price than the incumbent's one. Hence in this situation, cost symmetry may hinder collusion.

Section 2 presents the basic model where our industrial setting is depicted and sustainability of collusion is defined. In section 3, we study how cost-based access regulation affects the sustainability of collusion. Section 4 concludes. Proofs of Lemmata and Proposition are relegated to an Appendix.

## 2 Model

Consider an infinitely repeated and differentiated duopoly game where two firms compete in prices for supplying substitutable<sup>3</sup> goods. We assume the demand

<sup>1</sup>Among others, Berger (2005) and Behringer (2009) examines recently welfare effects of cost-based regulation in a two-way access model.

<sup>2</sup>In related framework, Collie (2004) discusses how demand elasticity impacts the sustainability of collusion. It is shown that the larger is the elasticity of demand, the easier it is to sustain collusion at the monopoly price.

<sup>3</sup>When goods are complements, collusion is not an issue.

system is linear and given by

$$\begin{aligned}d_1(p_1, p_2) &= 1 - b_1 p_1 + p_2 \\d_2(p_2, p_1) &= 1 - b_2 p_2 + p_1\end{aligned}$$

where  $b_i \geq 1$  is a parameter<sup>4</sup> accounting for the direct price elasticity of demand  $i = 1, 2$ . Indeed the price elasticity of demand for product  $i$  is defined as  $e_i = -\frac{\partial d_i(p_i, p_j)}{\partial p_i} \frac{p_i}{d_i(p_i, p_j)} = b_i \frac{p_i}{d_i(p_i, p_j)}$  so if  $b_i$  increases<sup>5</sup> then the price elasticity of demand goes up, if all things remain equal.

We consider a vertically related industry in which firm 1 (the incumbent) provides an essential input, for instance access to a network, to firm 2 (the competitor). Access is supposed to be charged at a given exogenous uniform price  $a \geq 0$ , the access charge. Profits are written respectively:

$$\begin{aligned}\pi_1(p_1, p_2) &= p_1 d_1(p_1, p_2) + a d_2(p_1, p_2) \\ \pi_2(p_2, p_1) &= p_2 d_2(p_2, p_1) - a d_2(p_2, p_1)\end{aligned}$$

All productions costs are normalized to zero.

In a stage game infinitely repeated, for a common discount factor  $\delta$ , firms maximize the present discounted value of future profits. Following Friedman (1971), collusion can be sustained as a subgame perfect equilibrium provided that the discount factor is sufficiently large. Using trigger strategies, firms revert to the Bertrand-Nash equilibrium as soon as a firm deviates from collusion. Thus, collusion at the joint profit-maximizing price is sustainable if the discount factor exceeds for both firms the critical value  $\delta_i$  defined as follows:

$$\delta \geq \delta_i = \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^*} \quad (1)$$

where  $\pi_i^c$ ,  $\pi_i^*$  and  $\pi_i^d$  represent respectively for a firm  $i$ , the collusion profit, the competition profit and the profit obtained by deviating from collusion. Hence condition (1), states that collusion at the joint profit-maximizing price is sustainable whether the present discounted value of profits from collusion, exceeds the profits obtained by deviating from collusion, followed forever thereafter by the Nash equilibrium profits.

We denote as  $\delta^* = \max\{\delta_1, \delta_2\}$  the *critical discount factor*. In the sequel we will pay special attention to the variations of this factor to the access price

<sup>4</sup>Notice that here  $\frac{1}{b_1 b_2}$  measures the degree of product differentiation, where products are independents if  $\frac{1}{b_1 b_2} \rightarrow 0$ .

<sup>5</sup>Indeed one can easily see that  $\frac{de_i}{db_i} = \frac{p_i(1+p_j)}{d_i(p_i, p_j)^2} > 0$ .

$a$ , as a result we will consider  $\delta^*$  as a function of  $a$ . Considering only interior solutions for all outcomes (i.e. competition, collusion and deviation), equilibrium demands for all outcomes must be always positive so that the following condition is valid:

$$b_2 \geq \max \left\{ \beta_1^d(b_1), \beta_2^d(b_1) \right\} \quad (2)$$

where functions  $\beta_i^d(b_1)$  are given in the Appendix and represent lower boundaries for an interior solution for deviation outcomes. Then in this industrial setting, the critical discount factor  $\delta^*(a)$  is defined in the following Lemma.

**Lemma 1** *When condition (2) holds, the critical discount factor is defined as*

$$\delta^*(a) = \max \left\{ \delta_1(a), \delta_2(a) \right\} \quad (3)$$

where

$$\delta_1(a) = \min \left\{ 1, \frac{(4B-1)^2 (2(B-1)a - 1 - b_1)^2}{A_1 A_2} \right\} \quad (4)$$

$$\delta_2(a) = \min \left\{ 1, \frac{(4B-1)^2 (2b_2(B-1)a - 1 - b_2)^2}{A_3 A_4} \right\} \quad (5)$$

with  $B = b_1 b_2 \geq 1$  and  $A_k, k = 1, 2, 3, 4$  are positive real expressions<sup>6</sup>.

Equipped with this definition, we are now considering how cost-based access pricing affects sustainability of collusion.

### 3 Cost-based access charge and incentives for collusion

In the sequel, we focus our analysis on the impact of access charge on sustainability of collusion in a neighborhood around cost-based regulation, i.e. when  $a$  tends towards 0. However, we do not question the optimality of such a regulatory practice since setting access prices to the cost of providing access has some well-known practical advantages<sup>7</sup> (see Armstrong and Sappington, 2007). First we investigate which individual critical values  $\delta_1$  or  $\delta_2$  is the maximum when cost-based access pricing applies (i.e.  $a = 0$ ) and second we determine how the critical discount factor  $\delta^*(a)$  varies with the access price  $a$ , again for  $a = 0$ .

<sup>6</sup>They are detailed in the proof of this Lemma.

<sup>7</sup>Remark that in our setting, cost-based access regulation is socially optimal if a break-even constraint is imposed on profits on access.

### 3.1 Sustainability of collusion

Let us first consider that the access price is cost-based regulated so that  $a = 0$ . We analyze how differences in direct elasticity parameters between firms (i.e.  $b_1$  and  $b_2$ ) modify the incentives for collusion for both competitors. More precisely we characterize the level of critical discount factor defined in (3) in the case where  $a = 0$  (i.e.  $\delta^*(0)$ ) and we compare both factors  $\delta_1(0)$  and  $\delta_2(0)$  according to the level of elasticity parameters  $b_1$  and  $b_2$ , where  $(b_1, b_2) \in [1, \infty)^2$ .

**Lemma 2** *The critical discount factor in a neighborhood around  $a = 0$  is*

$$\delta^*(0) = \begin{cases} 1 & \underline{\beta}(b_1) \geq b_2 \\ \delta_1(0) & \text{if } \underline{b}_1 \geq b_2 \geq \underline{\beta}(b_1) \\ \delta_2(0) & \bar{\beta}(b_1) \geq b_2 \geq \underline{b}_1 \\ 1 & b_2 \geq \bar{\beta}(b_1) \end{cases}$$

This Lemma 2 states that the critical discount factor corresponds to one of the firm which faces the most price-sensitive demand. That is to say, all things being equal, when a firm  $i$  faces a more elastic demand than its rival  $j$  (in the sense where the elasticity parameter  $b_i$  is higher than  $b_j$ ), then it has more incentives to deviate from the collusive agreement. This result is illustrated in figure 1. The non-shaded area corresponds to interior solutions for all outcomes considered (i.e. competition, collusion and deviation for each firm)

In order to give an intuition behind the result in Lemma 2, we can simply focus on how collusion gains<sup>8</sup> for a given firm ( $\pi_i^c - \pi_i^*$ ) are sensible to their own elasticity parameter  $b_i$ . In Lemma 2, the access price is assumed to be cost-based ( $a = 0$ ), then asymmetries between firms merely lie in demand parameters. To illustrate let consider that  $b_1 \geq b_2$  thus a higher direct elasticity of demand for firm 1 can be viewed as a strategic disadvantage, so its (competitive or collusive) profit is generically lower than firm 2's profit. Therefore an efficient collusive agreement will totally internalize this strategic disadvantage reducing the negative impact it could yield on the joint profit. Consequently, firm 1 will earn less profit than firm 2 if collusion is achieved. So collusion gains are lower for firm 1, that is to say firm 1 has more incentives to deviate from the collusive agreement when  $b_1 \geq b_2$ .

<sup>8</sup>Similar arguments can be produced for both deviation gains and cost punishments.

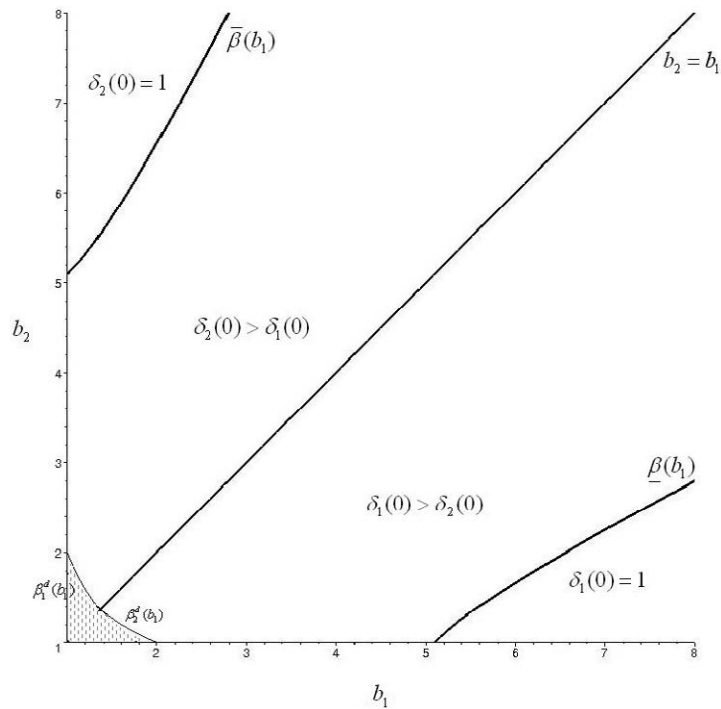


Figure 1: Critical discount factors for  $a = 0$  in the  $(b_1, b_2)$  plane

### 3.2 Comparatives statics

In this subsection, we investigate the following question: is a cost-based regulated input price a factor which hinders or facilitates collusion in differentiated and vertically related markets? More precisely, we determine the variations of the critical discount factor defined in (3) around  $a = 0$ , i.e.  $\frac{d\delta^*(0)}{da}$ . The following lemma can be stated.

**Lemma 3** *In the neighborhood of  $a = 0$ , the critical discount factor varies as follows:*

$$\frac{d\delta^*(0)}{da} \begin{cases} \geq 0 \\ \leq 0 \end{cases} \text{ if } \begin{cases} b_1 \geq b_2 \\ b_2 \geq b_1 \end{cases}$$

Lemma 3 shows how an increase in the access price (in a neighborhood around cost-based regulation) will either hinder or facilitate collusion in the industry depending on the differences of the elasticity parameters of demand. As stated in Lemma 2, remark that the slope of the critical discount factor

depends on which firm has more incentives to deviate from collusion and whether a firm faces *ex ante* a more elastic demand than its rival. To understand results stated in Lemma 3, we have now to appreciate how each critical threshold  $\delta_1$  and  $\delta_2$  is varying with the access charge  $a$  (around  $a = 0$ ). Again, to clarify the intuition we focus on collusion gains ( $\pi_i^c - \pi_i^*$ ) and on their variations for each firm. First, consider firm 1 which owns the essential input. An increase in the access price yields two well-known positive effects on firm 1's profits: a *direct revenue effect* and a *strategic price effect*. While the direct effect plays for both collusion and competition settings, the strategic effect totally vanishes when firms collude. Indeed, collusive prices are set to internalize this effect. Instead, with competition this strategic effect creates a competitive advantage that benefits the incumbent. As a result, an increase in the access price reduces collusion gains for firm 1 and then leads to more incentives to deviate from the collusive agreement. This simple intuitive argument is at the root of the result stating the critical factor  $\delta_1$  has an increasing shape with respect to  $a$ .

The same arguments prevail for firm 2 except that now both direct and strategic effects are negative since the access charge is a cost borne by this firm. Consequently, an increase in the access price improves collusion gains for firm 2 and thus creates more incentives to stick to the collusive agreement. This explains that the critical factor  $\delta_2$  has a decreasing shape with respect to the access charge.

Furthermore, the following proposition ensues from previous Lemmata.

**Proposition 1** *In a vertically related industry, cost-based regulation hinders the sustainability of collusion whenever the demand for the incumbent's good is less price sensitive than the demand for the competitor (and conversely).*

As Lemmata 2 and 3 state, when the competitor's demand is more price sensitive than the incumbent's one, the critical discount factor is defined by  $\delta_2(a)$  and is decreasing with respect to  $a$ . Therefore, the competitor has less incentive to collude whenever the access price is cost-based. Accordingly, cost-based access regulation yields more cost symmetry in the industry thus, the result in Proposition 1 means that it may be harder to collude among equals in markets. This can be viewed as an argument against the conventional doctrine.

Such a result is especially significant for network industries where the price sensitivity condition considered in Proposition 1 is a stylized fact commonly accepted to represent switching costs for consumers. Indeed an extended theo-

retical and empirical economic literature<sup>9</sup> shows that switching costs are higher for consumers who address their demand to incumbents. This can be represented in terms of differences between direct price sensitivities favoring the incumbent (i.e. in our model  $b_1 < b_2$ ).

We close this section with an heuristic discussion about what should be the regulatory response in such an environment. From a regulatory point of view<sup>10</sup>, constraints (1) should be defined as  $\delta \leq \delta^*(a)$  which corresponds to an *anti-collusive constraint* that should be taken into account in order to determine the welfare maximizing access charge. Indeed, for a given level of the discount factor  $\delta$ , the critical factor, defined in Lemma 2, should be settled up at a sufficiently high level in order to prevent sustainability of collusion in the industry. Therefore, our previous results might be useful to reconsider optimal access regulation since it has been proved that cost-based regulation would prevent or enhance sustainability of collusion. For instance when  $b_1 < b_2$ , no conflict occurs between access regulation and competition policy, for moderately low values of the market discount factor (i.e.  $\delta < \delta^*(0)$ ). In this case the anti-collusive constraint is slack, and with cost-based access regulation firms have not incentives to collude. On the other side, for higher values of  $\delta$  (i.e.  $\delta > \delta^*(0)$ ), the regulator might allow an access mark-up in order to make sustainability of collusion more arduous. As the anti-collusive constraint is now binding in this case, the regulator should state an access charge  $a^* > 0$  defined by  $\delta^*(a^*) = \delta$ , since  $\delta^*(a)$  is decreasing around  $a = 0$ . This access charge would trade-off optimal access regulation and collusion concerns.

## 4 Conclusion

Our paper states an interesting result considering market for utilities (telecommunications, energy, transports) for which access to an essential facility is crucial to promote competition. Generally, in these industries, incumbents have competitive advantages which not depend on prices such as goodwill, brand effects or installed base of consumers. It is well known that such advantages give rise to switching costs, that can lead to more price sensitivity of demand for new competitors (i.e. higher elasticity of demand). Applying our result in this context means that cost-based access regulation may weaken sustainability of collusion

<sup>9</sup> Among others Klemperer (1987), Knittel (1997) and Shy (2002).

<sup>10</sup> However it might be questioned whether regulatory structures should be in charge of the control of collective dominance.

in the industry.

## Appendix

• **Proof of Lemma 1.** In order to prove this Lemma we proceed by determining competitive, collusion and deviation outcomes in turns.

(a) **Price competition.** The competitive outcome is given by the Bertrand-Nash equilibrium where each firm maximizes its profits, given the price of its competitor. Routine calculations yield (interior) equilibrium price and profits of the firms as functions of the access charge  $a \geq 0$ :

$$\begin{aligned} p_1^* &= \frac{3b_2}{4B-1}a + \frac{1+2b_2}{4B-1} \text{ and } p_2^* = \frac{2B+1}{4B-1}a + \frac{1+2b_1}{B} \\ \pi_1^* &= -\frac{b_2(8B+1)(B-1)}{(4B-1)^2}a^2 + \frac{1+b_2+8B(B+b_2)}{(4B-1)^2}a + \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} &+ \frac{(1+2b_2)^2 b_1}{(4B-1)^2} \\ \pi_2^* &= b_2 \left( \frac{2(B-1)a - (1+2b_1)}{4B-1} \right)^2 \end{aligned} \quad (\text{A.2})$$

where  $B = b_1 b_2 > 1$ . In this linear setting, equilibrium demands must be non-negative with the above price collection so we restrict our attention to interior solutions for which  $a \leq a^* = \frac{1+2b_1}{2(B-1)}$  where  $a^*$  is the value of the access charge for which  $d_2^* = 0$ .

(b) **Collusion.** When firms collude, they are assumed to behave as a cartel that maximizes total industry profits. It is straightforward to calculate the joint profit-maximizing prices and the corresponding profits for each firm:

$$\begin{aligned} p_i^c &= \frac{1+b_i}{2(B-1)} \text{ with } d_i^c = \frac{1}{2} \\ \pi_1^c &= \frac{2(B-1)a + 1 + b_2}{4(B-1)} \text{ and } \pi_2^c = \frac{1 + b_1 - 2(B-1)a}{4(B-1)} \end{aligned} \quad (\text{A.3})$$

For an interior outcome to be valid, we must also verify the feasibility condition  $a \leq a^c = p_2^c$  where  $p_2^c - a^c = 0$ . Notice that  $a^c < a^*$ . Therefore whenever  $a > a^c$  the collusion profit for firm 2 is zero and it has no incentives to collude. As a result using (1),  $\delta_2 \geq 1$  and moreover  $\delta_2 = 1$  when  $a > a^*$ .

(c) **Deviation.** If a given firm deviates from collusion in any stage of the game, then it maximizes its profits in that stage given that the other firm produces

the collusive price. Therefore, the price and profits of the deviating firm are

$$\begin{aligned} p_1^{d_1} &= \frac{1}{2b_1}a + \frac{2(B-1)+1+b_1}{(4B-1)b_1} \text{ and } p_2^{d_2} = \frac{1}{2}a + \frac{2(B-1)+1+b_2}{(4B-1)b_2} \\ \pi_1^{d_1} &= \frac{1}{4b_1}a^2 + \frac{2Bb_1-3b_1-1}{(4B-1)b_1}a + \frac{1}{b_1} \left( \frac{2B-1+b_1}{4B-1} \right)^2 \end{aligned} \quad (\text{A.4})$$

$$\pi_2^{d_2} = \frac{b_2}{4}a^2 - \frac{2B-1+b_2}{4B-1}a + \frac{1}{b_2} \left( \frac{2B-1+b_2}{4B-1} \right)^2 \quad (\text{A.5})$$

For equilibrium demands to be nonnegative with this price collection, we must verify that  $a \geq \max\{a_1^d, a_2^d\}$  where  $a_1^d = a^c - b_1$  and  $a_2^d = a^c - 1 - \frac{1}{2b_2}$  are values of the access charge for which  $d_1^{d_2} = 0$  and  $d_2^{d_1} = 0$  respectively. However one can note that for relatively high values of  $(b_1, b_2)$ , thresholds  $a_i^d$  are below zero and deviation demands are positive for all  $a$ . More precisely this is the case when condition (2) in the text holds, where  $\beta_1^d(b_1) = \frac{3+\sqrt{9+8b_1}}{4b_1}$  and  $\beta_2^d(b_1) = \frac{3b_1+1}{2b_1^2}$  with  $\beta_2^d(b_1) \geq \beta_1^d(b_1)$  for  $b_1 \geq \frac{1+\sqrt{3}}{2}$ . Condition (2) entails that demands for all outcomes are always positive.

**(d) Critical discount factors.** When (2) holds, using (1), (A.1), (A.2), (A.3), (A.4), (A.5) and considering that discount factors are bounded above by one, we obtain (4) and (5) in the text with  $A_1 = \left[ (4B-1)^2 - 6B \right] [2a(B-1) - 1] - b_1(8B-5)$ ,  $A_2 = 2(B-1)(2B+1)a - (1+3b_1+2B)$ ;  $A_3 = 2(B-1)(8B-5)b_2a - \left[ (4B-1)^2 - 6B \right] - b_2(8B-5)$  and  $A_4 = 6(B-1)b_2a - (1+3b_2+2B)$ . ■

• **Proof of Lemma 2.** Setting  $a = 0$  in (4) and (5), we define two bounds  $\bar{\beta}(b_1) = \frac{4b_1^2 - 8b_1 - 1 + (2b_1 + 1)\sqrt{4b_1^2 - 20b_1 + 1}}{16b_1}$  and  $\underline{\beta}(b_1) = \frac{2b_1^2 + 2b_1 + 1 + (2b_1 + 1)\sqrt{b_1^2 + b_1 + 1}}{2b_1}$  for which  $\delta_1(0) \leq 1$  if  $b_2 \geq \underline{\beta}(b_1)$  and  $\delta_2(0) \leq 1$  if  $b_2 \leq \bar{\beta}(b_1)$ . Moreover one can easily remark that  $\bar{\beta}(b_1) < b_1$  and  $\underline{\beta}(b_1) > b_1, \forall b_1 > 1$ . Hence direct calculations show that

$$\delta_1(0) - \delta_2(0) = \left[ 4(4B-1)^2(B-1) \frac{(1+b_1)^2}{b_2 B_1 B_2 B_3 B_4} B_5 \right] (b_1 - b_2)$$

with  $B_k = A_k < 0$ , for  $k = 1, 2, 3, 4$  and when  $a = 0$ , and  $B_5 = 8B^3 + (32b_2 + 8b_2^2 + 5)B^2 + (5b_2^2 - 13b_2 + 5)B - 4b_2^2 - b_2 > 0$  for all  $(B, b_2) \in [1, \infty)^2$ . Hence  $\delta_1(0) - \delta_2(0)$  has the same sign as  $(b_1 - b_2)$  whenever  $\delta_1(0) \leq 1$  and  $\delta_2(0) \leq 1$ . ■

• **Proof of Lemma 3.** Assume first that  $b_1 \geq b_2$  so that  $\delta_1(0) \geq \delta_2(0)$ . Defining bivariate real functions  $C(\cdot), D(\cdot)$  and  $E(\cdot)$  such that for all  $(x, y) \in [1, \infty)^2$ ,  $C(x, y) = 1 + 3x + 2xy$ ;  $D(x, y) = 1 + 16(xy)^2 + 8x^2y - 14xy - 5x$  and  $E(x, y) = 16(xy)^2 + 16x^2y - 6xy - 7x - 1$ . Notice  $E(x, y)$  is positive in

$(x, y) \in [1, \infty)^2$  since  $E(1, 1) = 18$ ,  $\frac{\partial E(x, y)}{\partial x} = 2(y + 1)(16xy - 3) - 1 > 0$  and  $\frac{\partial E(x, y)}{\partial y} = 2x(8x + 16xy - 3) > 0$ . Then

$$\frac{d\delta_1(0)}{da} = \frac{8(4B - 1)^2(1 + b_1)b_1(B - 1)^2}{C(b_1, b_2)^2 D(b_1, b_2)^2} E(b_1, b_2) \quad (\text{A.6})$$

then a glance at (A.6) yields  $\text{sgn}\left(\frac{d\delta_1(0)}{da}\right) = \text{sgn}(E(b_1, b_2)) > 0$ .

Assume now  $b_1 < b_2$  so that  $\delta_2(0) < \delta_1(0)$  then

$$\frac{d\delta_2(0)}{da} = -\frac{8(4B - 1)^2(1 + b_2)b_2(B - 1)^2}{C(b_2, b_1)^2 D(b_2, b_1)^2} E(b_2, b_1)$$

which leads to conclude that  $\text{sgn}\left(\frac{d\delta_2(0)}{da}\right) = -\text{sgn}(E(b_2, b_1)) < 0$ . ■

• **Proof of Proposition 1.** Straightforwardly from Lemmata 2 and 3. ■

## References

- [1] Armstrong, M. and D. Sappington, 2007. Recent Developments in the Theory of Regulation, in: M. Armstrong and R.H. Porter, eds., Handbook of Industrial Organization, vol. 3, North Holland, 1557-1700.
- [2] Behringer, S., 2009. Entry, access pricing, and welfare in the telecommunications industry, *Economics Letters*, 102(3), 185-188.
- [3] Berger, U., 2005. Bill-and-keep vs. cost-based access pricing revisited, *Economics Letters*, 86, 107-112.
- [4] Collie, D.R., 2004. Collusion and the elasticity of demand, *Economics Bulletin*, 12(3), 1-6.
- [5] Collie, D.R., 2006. Collusion in differentiated duopolies with quadratic costs, *Bulletin of Economic Research*, 58:2.
- [6] Deneckere, R., 1983. Duopoly supergames with product differentiation. *Economics Letters* 11, 37-42.
- [7] Deneckere, R., 1984. Corrigenda. *Economics Letters* 15, 385-387.
- [8] Friedman, J.W., 1971. A noncooperative equilibrium for supergames, *Review of Economic Studies*, 38, 1-12.

- [9] Ivaldi, M., B. Jullien, P. Rey. P. Seabright and J. Tirole, 2003. The Economics of Tacit Collusion. IDEI Working Paper, n°222, Report for DG Competition, European Commission.
- [10] Klemperer, P., 1987. Markets with consumer switching costs, *Quarterly Journal of Economics*, 102, 375–394.
- [11] Knittel, C., 1997. Interstate long distance rates: Search costs, switching costs, and market power, *Review of Industrial Organization*, 12, 519–536.
- [12] Rothschild, R., 1992. On the sustainability of collusion in differentiated duopolies, *Economics Letters* 40, 33–37.
- [13] Shy O., 2002. A quick-and-easy method for estimating switching costs, *International Journal of Industrial Organization*, 20, 71–87.