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Partial Horizontal and Vertical Ownership

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Partial Horizontal and Vertical Ownership

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Abstract

Others have shown that in vertically related Cournot oligopolies partial ownership could have no real effects on total output or price choice, and in a separate way that increasing cross ownerships among rivals leads to more collusive outcome. In a complementary manner we study the interactions between vertical and horizontal partial ownerships giving no control over target. This paper shows that when the choice of optimal cross ownership profile is simultaneous, a mixed equilibrium with upward vertical and horizontal participations can be achieved, vertical and horizontal ones being strategic substitutes. We finally exhibit the significant influence of vertical participations on output price as on profits and on consumer surplus, which in our model is harmful at optimum from a consumer and a social point of view.

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1 Introduction

Cross ownership are frequently held as financial arrangements in vertically related industries, these equity trading giving no control over the target are known in the literature as "silent" financial interests. Partial Vertical ownership (PVO) are contracted for several reasons: to align the interest of the target and the acquirer in a same goal, to reduce transaction costs, to acquire information, to reduce double marginalization, or at contrary to enhance market power through foreclusion. Except models with few strong assumptions as in Flath (1989)[1], literature on the task; see Greenlee and Raskovich (2006); states that PVO in Cournot successive oligopolies with homogeneous goods, constant marginal costs, and linear demand may have no effects on output choices. In his principal-agent model Riordan (1991) states that backward partial integration in a monopsony industry has an ambiguous effect on output production. On one hand the partial acquisition through cost sharing curtails incentives of the downstream firm to act monopsonistically, but on the other hand it undermines incentives of upstream firms to reduce costs. Results for Partial Horizontal Ownership (PHO) coincide (Reynolds and Snapp, 1986; Reitman, 1994) with those found in the merger literature as in Salant, et al., (1983); when partial horizontal ownership equilibrium among Cournot firms is possible[2]; i.e. reduced output production through lessened competition leads to greater prices and firms profits but leads also to a fall of the consumer surplus and of the total welfare. Nevertheless in a Cournot model if synergy gains are possible, as size effects of capital[3], Farrel and Shapiro (1990) show an improvement in industry performance and welfare. In dynamic models studies, Malueg (1991) affirms that in most cases [4] collusion is more likely to occur when there is a net of shareholdings between firms, still dynamic but in a Bertrand framework the model of Gilo, et al., (2006) shows increased likelihood of tacit collusion when firms are linked by partial cross ownership.

[1] In Flath (1989) we can find achievement of vertical integration through upward partial ownership under assumptions on production (Leontief) and on demand (log convex with constant elasticity) functions.

[2] PHO or mergers equilibria indeed are not generally formed with three or more firms competing à la Cournot due to the "outsider" effect.

[3] Additional capital reduces marginal cost.

[4] Under certain conditions on demand parameters, making the demand function convex.

Our work attempts to combine these two types of equity participations in a single model in order to study their joint effects on the strategic behavior of firms. In addition to the existing models, this article leads to original results concerning the impact of PVO, particularly due to the union of horizontal and vertical equity interests choices in a same model. Indeed the perspective of being partially vertically integrated decreases downstream firms incentives to acquire PHO. More precisely, this paper highlights a strategic substitutability between PHO and PVO, increasing PVO reduces PHO and vice versa. PHO being harmful to competition, reducing it appears to be good news from a social point of view. But our analysis suggest that this loss in horizontal collusion is completely offset by the acquisition of PVO, leading to the conclusion that in our industry holding PVO may have the same effects on performance and welfare than acquiring PHO. We thus find a significant (positive or negative) distortion effect of upward PVO on output. Moreover under some conditions on the industry structure[5], an optimum[6] combining vertical and horizontal participations exists as a result of the maximization of equilibrium profits, it allows (as in an only PHO case[7]) to attain the monopoly level of production and profit. Until now literature on partial ownership has considered that PVO has no effect on production and on surplus but our research exposes a more pessimistic point of view, these results should raise antitrust concerns about vertical passive investments in successive oligopolies.

2 Set-up

The model is a Cournot-Nash one of successive oligopolies linked by the input supply and by financial arrangements as PVO and PHO. Upstream firms (manufacturers) produce a homogeneous intermediate good, which serve as an input in the production of a final good by downstream firms (retailers). Each unit of final good requires one unit of intermediate good, using a one-to-one technology, hence total quantity on the intermediate market must equate total quantity on the final market, $R = \sum_{k=1}^{m} r_k = \sum_{i=1}^{n} q_i = Q$.

For simplicity we assume that marginal costs of production, transformation and retail are constant. To alleviate the notation, we standardize these costs to 0. We also suppose that the demand curve is linear, $p_F = 1 - Q$.

It is useful to detail the step of partial ownership (PO=PHO+PVO) choice:

[5] The number of upstream firms has to be greater (except for $m = n = 2$ and 3) to the number of downstream firms for an equilibrium of PO to be relevant in the range of intervals where non controlling PO should normally lie. These intervals are detailed in the fourth section.

[6] We use in this paper the term equilibrium for the results of maximization in the 2^{nd} stage of our game, and we call optimum the results of the maximization in the 1^{st} stage of our three-stage game. Equilibrium quantities will be annotated with a $^+$ and optimum ones with a *.

[7] Refer to Reynolds and Snapp (1985), theorem 2.

The downstream firms simultaneously choose to enter or not in the capital of rivals firms through PHO. Furthermore upstream suppliers decide simultaneously to acquire (or not) shares in downstream retailers. These partial equity interests give "residual rights[8]" on the redistribution of profits. As these PO do not give control over the target firm, they are silent financial interests. Each firm, as an owner-manager[9] firm, both target and purchaser, maximizes the operating (its revenue minus its costs) profit in its entirety. In addition it takes into account the shares bought and sold[10]. We assume that the acquisition of shares (equity) is through an holding or through an investment fund (wich is independent of the firms and of the industry we consider) which pays or receives transfer prices in exchange of shares of capital. So does this participation only right to a partial redistribution of operating profits of the target[11]. Finally, we believe that these investments are made before production decisions, and therefore they place in the 1st stage of our game.

Table I. Notation

u_k	upstream producers
d_i	downstream retailers
m	number of firms on the upstream market
n	number of firms on the downstream market
r_k	output of intermediate good firm k
q_i	output of final good firm i
Q	total output
p_I	intermediate good price
p_F	final good price
h_{ij}	the ith firm's partial horizontal ownership interests in the jth firm (downstream)
v_{ki}	the kth firm's partial vertical ownership interests in the ith firm (upward)

In the 2nd stage, producers maximize their profits on the intermediary market by choosing the uniform sale price of the input needed to manufacture the final product. In the 3rd stage, dealers then select the quantities to put on the market considering the final price of input as given. All financial arrangements are set and are considered by all firms as fixed in the 2nd and the 3rd steps.

We solve backward, to obtain a subgame perfect Nash equilibrium (SPNE).

[8] "Residual" rights, the term refers to the theory of "residual rights" partly fueled by Grossman and Hart (1986). Here it is a partial rebate of ex-post profits of the firm target.

[9] The company is both manager and principal shareholder. Thus we avoid conflicts of interest.

[10] The stakes being taken to a percentage, there is colinearity between profit as a whole and a fraction of the profit, which means no change in choosing quantities. Nevertheless, the share held by competitors of a particular firm will have an influence in its choice of optimal PO.

[11] In exchange of a transfer price paid to the holding outside of the industry.

3 Successive Oligopolies with Horizontal and Vertical Partial ownership

3.1 Derived demands (2^{nd} and 3^{nd} stages)

3.1.1 Retailers

We begin with the downstream industry, determining the derived demand that faces firms in the upstream industry. Given all existing partial ownership arrangements, retailer i sets output of the final good q_i to maximize its payoff:

$$\pi_i = (1 - \sum_{j \neq i} h_{ji} - \sum_{k=1}^{m} v_{ki})\pi_i^{op} + \sum_{j \neq i} h_{ij}\pi_j^{op} \tag{1}$$

Upstream operating profit is provided by the sale of the input, and downstream operating one by the sale of the final good:

$$\pi_k^{op} = r_k p_I \tag{2}$$

$$\pi_i^{op} = q_i(p_F - p_I) \tag{3}$$

Substitute (3) in (1) to obtain the developed form:

$$\pi_i = (1 - \sum_{j \neq i} h_{ji} - \sum_{k=1}^{m} v_{ki})q_i(p_F - p_I) + \sum_{j \neq i} h_{ij}q_j(p_F - p_I) \tag{4}$$

$$\pi_i = (1 - \sum_{j \neq i} h_{ji} - \sum_{k=1}^{m} v_{ki})q_i(1 - \sum_{i=1}^{n} q_i - p_I) + \sum_{j \neq i} h_{ij}q_j(1 - \sum_{i=1}^{n} q_i - p_I) \tag{5}$$

First pose the profit second derivative to quantities, $\frac{\partial^2 \pi^{D^i}}{\partial {q_i}^2} < 0$, the sign of the previous expression is easily verifiable[12], this second-order condition is sufficient to ensure the concavity of the profit function with respect to quantities.

Presently the first-order for maximization of π^{D^i} with respect to q_i, is

$$2q_i = 1 - \sum_{j \neq i} q_j - p_I - \frac{\sum_{j \neq i} h_{ij}q_j}{1 - \sum_{j \neq i} h_{ji} - \sum_{k=1}^{m} v_{ki}} \tag{6}$$

Now condense this by imposing symmetry that will hold in equilibrium: let q be the equilibrium final good output. Also define $\alpha_i = \sum_{j \neq i} h_{ji}$ to be the total

[12] See appendix.

fraction of firm i's profit owned by other downstream firms, $\beta_i = \sum_{j \neq i} h_{ij}$ to be the total shares of other downstream firms' profits owned by firm i, and $\gamma_i = \sum_{k=1}^{m} v_{ki}$ to be total shareholdings in firm i's profit held by all upstream firms. Conditions of existence[13] allow us to pose $\alpha_i = \alpha_j = \alpha$, $\beta_i = \beta_j = \beta$, and $\gamma_i = \gamma_j = \gamma$, for all i, j and k. Solving (6), we get the expression for equilibrium final good output in terms of the intermediate good price and PO,

$$q = \frac{(1-p_I)(1-\alpha-\gamma)}{(n+1)(1-\alpha-\gamma)+\beta} \tag{7}$$

Since one unit of intermediate good input is required for each unit of final good output, the derived demand facing the intermediate good market is

$$nq = Q = n\frac{(1-p_I)(1-\alpha-\gamma)}{(n+1)(1-\alpha-\gamma)+\beta} \tag{8}$$

Solve (8) for p_I to obtain the derived inverse demand facing producers on the intermediate good market,

$$p_I = \frac{(n+1)(1-\alpha-\gamma)Q+\beta Q+n(\alpha+\gamma-1)}{n(\alpha+\gamma-1)} \tag{9}$$

3.1.2 Producers

Producer k on the upstream market maximizes its payoff given all PO:

$$\pi_k = \pi_k^{op} + \sum_{i=1}^{n} v_{ki}\pi_i^{op} \tag{10}$$

Substitute (2) and (3) in (10); technology of transformation being one to one, one unit of output requires one unit of input; replace $Q = R$, to get an expanded form of producer k profit:

$$\pi_k = r_k p_I + \sum_{i=1}^{n} v_{ki}(1-R-p_I) \tag{11}$$

Substituting (7) and (9) in (11) and maximizing with respect to r_k, gives the first-order condition. We then define $\delta_k = \sum_{i=1}^{n} v_{ki}$ to be the total stakes in all downstream firms' profits owned by firm k through upward PVO. Conditions of

[13] $\frac{\partial \pi_i}{\partial q_i} = 0$, $\frac{\partial \pi_j}{\partial q_j} = 0$, thus $\frac{\partial \pi_i}{\partial q_i} = \frac{\partial \pi_j}{\partial q_j}$.

existence again allow us to pose $\delta_k = \delta_l = \delta$ for all i, k and l We condense and we impose symmetry that will hold in equilibrium for input quantities r,

$$r = \frac{n^2(\alpha+\gamma-1)}{\substack{n[1-\alpha-\gamma+n+\beta-n\alpha-n\gamma+m(1+n-\alpha-\gamma+\beta-n\alpha-n\gamma)] \\ +2m\delta(\alpha+\gamma-\beta-1)}}$$

We thus obtain the expression for equilibrium intermediate good in endogenous terms of PO only. This expression is easily simplifying considering simple economics intuitions for the symmetric case. So it is reasonable to think that the sum of horizontal participations bought for example by one downstream equals the sum of horizontal participations sold by that same firm. In that case we now note $\alpha = \beta = H_d$. In contrast, for vertical equity interests the sum of PVO bought by an upstream firm do not necessarily[14] matches the sum of PVO sold by a downstream firm, but the relation $\gamma = \frac{m}{n}\delta$ holds at equilibrium[15] and thus we are able to pose $\gamma = V_d$, $\delta = V_u$, and $V_d = \frac{m}{n}V_u$. We now derive the equilibrium input quantity r^+, in terms of decisions parameters only:

$$r^+ = \frac{n^2(nH_d+mV_u-n)}{\substack{n[n^2H_d-n^2-n+m(3V_u+nV_u-n+mV_u+nmV_u+n^2H_d-n^2)] \\ -2m^2{V_u}^2}} \tag{12}$$

From (12), we get total input and output quantity $R^+ = mr^+ = Q^+$ and so we are able to derive input and output prices, equilibrium profits π_k^+ and π_i^+, only in terms of PO the last remaining decisions variables.

We now proceed to a comparative static on equilibrium quantities in the goal of highlighting the effects of partial ownership on aggregate quantity of output and on the whole industry payoffs, we also derive as follow the best response function of an upstream and a downstream firm in order to study the strategic interaction between PVO and PHO.

4 Comparative Static

Until now only PHO have shown a real impact in research works, we start by studying theirs strategic interactions with PVO.

Solving first order condition $0 = \partial\pi_i^+/\partial H_d$ for H_d, one obtains a downstream firm's reaction function R_i, which gives for each value of PVO (V_u) chosen by an upstream firm k, the value of PHO representing firm i's best response:

$$R_i(V_u) = \frac{n[n^2-n+m(n^2-n+3V_u-nV_u+mV_u-nmV_u-2m{V_u}^2]}{n^3(1+m)} \tag{13}$$

[14] Except in the case where the number at each level of the industry is the same, i.e. $m = n$.

[15] The sum of all vertical shares bought must equal the sum off all vertical shares sold, this notes $m\delta = n\gamma$.

$$\frac{\partial R_i(V_u)}{\partial V_u} = \frac{m[-4mV_u + n(3 + m - n - mn)]}{n^3(1 + m)} < 0 \tag{14}$$

Solving first order condition $0 = \partial \pi_k^+ / \partial V_u$ for V_u, one obtains an upstream firm 's reaction function R_k, which gives for each value of PHO (H_d) chosen by an upstream firm k, the value of PVO representing firm k's best response. For notational simplifications purposes we only derive the sign of variation of R_k subject to a change in H_d, $\frac{\partial R_k(H_d)}{\partial H_d} < 0$. The derivatives of the two reaction functions being negative, they ensure that the PHO and the PVO vary in the opposite direction. In other terms PHO and PVO are strategic substitutes, say for example when the value of PVO increases the amount of PHO decreases. This strategic effect is a first step to characterize the influence of PVO on control variables resulting from profits maximization (e.g. output prices, input quantity, etc.) and on welfare indicators (e.g. consumer surplus, social welfare, etc.). In fact, the PHO have a negative effect on production, the fact of reducing its PHO holdings indirectly increases the amount of output production.

As a second step we derive comparative to prove the direct impact of PVO on final good quantities, intermediate price and industry profits.

$$\frac{\partial Q^+}{\partial V_u} = \frac{n^2m^2(-n^2mH_d - 3n^2H_d + 4nmH_dV_u + 2n^2 - 4nmV_u + 2m^2{V_u}^2)}{Z^2} \tag{15}$$

The sign of the denominator of (14) being obviously positive, the Z parameter is set to replace a much longer third degree polynomial in the purpose of simplification. We suppose $m \geqslant 2$ and $n \geqslant 2$ for the game to exist, thus the sign of this expression is either positive or either negative depending on the possible amounts of PO (V_u and H_d).

In this regard it is interesting to characterize the values of equity transactions that correspond to the case of "silent" or "passive" investments. In the next section we find optimal values of vertical and horizontal ownership resulting from the maximization of the equilibrium profits either subject to V_u for π_k^+ either subject to H_d for π_i^+. This choice of PO is endogenous, and we hope that acquired PVO lie in the interval $I = [0, \frac{n}{l+1}]$ and that acquired PHO lie in $J = [0, \frac{n-1}{n}]$. The choice of these intervals is in line with the reality of equity transactions on regulated markets. Indeed, the upper limit for PVO and for PHO allows a target firm to retain at least a "relative" majority of its capital, if these limits are exceeded we can no longer speak of non-controlling shareholdings. Thus we want the acquired firm to keep control[16] of its decisions.

$\frac{\partial Q^+}{\partial V_u} > 0$, $if\ H_d < \frac{-2(n^2 - 2nmV_u + l^2{V_u}^2)}{n(-n-nm+4mV_u)}$, and $\frac{\partial Q^+}{\partial V_u} < 0$ in the opposite case. These two possibilities are consistent with the fact that PO should lie in the

[16] It is not always true that holding the majority of financial interests gives corporate control. Indeed there are companies that have relatively complicated financial and governance structures, in which there is a distinction between voting and non-voting securities. A firm holding non-voting stocks has financial interests (partial rebate of profit) but no corporate control. In our model the equity shares sold can be either voting stocks either non-voting stocks, but only either one or the other for the PO choices in the whole model.

intervals I and J defined sooner. This proves that PVO play a true role in the output decision, PVO can either increase either decrease the output production, the sense of change depending on the quantity of PHO in the downstream industry. Optimum values of PO being chosen (at the 1^{st} stage of the game) before equilibrium quantities (3^{rd} stage), at optimum[17] H_d^* is greater than the threshold just calculated before, PVO thus lead to a reduction of the final good production and this way PVO increase incentives to collude by lessening competition in the downstream market. The PVO effect in this case is similar to a PHO effect ($\frac{\partial Q^+}{\partial H_d} < 0$ for value of V_u lying in I and J) in the sense of variation, but it may differ in the magnitude. We establish in the next section a table containing the main indicators of surplus and welfare in three different situations in the goal of comparing the effects of the PO on these values.

Before deriving the optimum amount of PO in the fourth section, we should look at the consequences on total industry profits of an increase in PVO. Obviously if PVO have a negative effect on market profits, they may only have adverse effects on economic welfare. We pose $\pi_u^+ = \sum_{k=1}^{m} \pi_k^+$ as the sum of all upstream profits, $\pi_d^+ = \sum_{i=1}^{n} \pi_i^+$ as the sum of all downstream profits, and $\Pi^+ = \pi_u^+ + \pi_d^+$ denotes the whole industry profit.

$\frac{\partial \Pi^+}{\partial V_u} > 0 \; if \; \frac{-2(mH_d-n)^2}{n(-3n-nm+4mV_u)} < H_d < \frac{-(2mV_u-n^2-n+n^2m-nm)(mV_u-n)}{n^3(m-1)}$, H_d is situated between two terms lying themselves in J. H_d could be smaller than the first term or greater than the second and being still included in J and in this case $\frac{\partial \Pi^+}{\partial V_u} < 0$. Nevertheless, at optimum the value of H_d^* lead us to believe that PVO have a positive influence on overall industry profits. Intuitively, this can be explained by the following: first output production being contracted by the acquisition of PVO and PHO, final price increases. Second the demand for the intermediate good (the input) is reduced, in consequence input prices get smaller and thus the margin of downstream retailers raises while upstream margin slightly decreases, however the value of the sum of the two margins grows. Finally upstream manufacturers recapture increased downstream profits through PVO.

We resolve to follow the optimal choice of PO, then we are able to obtain the main indicators of welfare and efficiency that we collect in the table II.

4.1 Optimal Class of Partial Ownership Profile (1^{st} stage)

4.1.1 Overview

A PO equilibrium is traditionally defined in the literature[18] as a network of PO within an industry such that, given the subsequent profits from product market

[17] Maximization of equilibrium profits subject to PO is exposed in the next section.
[18] Reitman (1994).

competition, all PO are individually rational for all firms, given the other PO in the market.

We are now recalling the timing and the structure of the first stage of the game, i.e. the PO equilibrium choice. Suppliers simultaneously choose the amount to invest in their downstream clients, in the same time the latter decide to invest horizontally in other downstream competitors. Partial ownership interests are frequently used on markets, but in our model where there is no entry (and no exit), upstream and downstream "owners-managers" choose the class of ownership profile in 1^{st} stage and once for all. This reflects the stylized fact that equity interests of a company adjusts less often than quantity and other short-term variables.

4.1.2 Equilibrium Choice of PO

The producer k has the option to take shares in its downstream clients, we assume that the equilibrium upstream profit function π_k^+ is concave in V_u. In addition the retailer i has the option to acquire horizontal shareholdings, we also assume that π_i^+ is concave[19] in H_D

In order to proceed U_k maximizes π_k^+ with respect to V_u, and simultaneously D_i maximizes π_i^+ with respect to H_D

$$\{\underset{V_u}{max}\ \pi_k^+; \underset{H_d}{max}\ \pi_i^+\}$$

The simultaneous resolution of this maximization problem gives the best value of PHO and of PVO chosen by in the final market V_u^* and H_d^*[20]. H_d^* do not reach the amount required ($0 < H_d^* < \frac{n-1}{n}$ the upper limit of the interval J) to achieve the monopoly level of collusion if only PHO were allowed[21], but in the same time $V_u^* > 0$. The presence of PVO replaces the fall in the amount of PHO at optimum, and this way it allows the industry to complete the maximum level of collusion by reaching the monopoly level of profit. This strategic substitutability already highlighted in the previous section can be now explained with intuitions. PVO; as PHO; lead to a raise in output price, this decreases the level of PHO required to achieve a particular level of collusion. Moreover PVO (via the perspective of a partial rebate of downstream profits) encourage upstream firms to decrease their input price in the purpose of expanding dealers margin. Consequently retailers now battle under a weaker competition conditions and they do not need anymore to acquire as much PHO as in a situation with no PVO.

We now precise the conditions under which these values are consistent with the definition of a "silent" investment, i.e. conditions under which the optimal

[19] See appendix for the study of second order conditions of profits functions.

[20] Explicit values of V_u^* and H_d^* being extremely long to derive, and not very important for the reasoning in this paper, we only give value of V_u^* in the appendix. We can provide the expression of H_d^* on request.

[21] Optimal values in this situation are derived in Table II in section 6.

PO lie in the intervals I and J. The structure of the whole industry is the critical one, first the number of firms at each level of the industry has to be greater than one for a successive oligopoly to exist. Second the number of upstream firms has to be greater ($m > n$ except for $m = n = 2$ and 3) to the number of downstream firms for the nonnegativity constraint of PO to be respected, especially for PVO. In other terms manufacturers only want to take (positive) vertical shares when the retailers' level is more concentrated than their own level of the industry. This phenomenon can be explained by the fact that the loss in upstream incomes is more than totally compensated by the redistribution (via PVO) of downstream profits only when downstream profits are relatively great, i.e. when the downstream industry is concentrated.

4.2 Regulation using a fixed threshold of participations

We present here a way for authorities to socially regulate the use of partial equity interests. Imagine for the antitrust agencies (i.e. FTC, DOJ, European commission, etc.) the possibility of limiting the maximum value of PVO than upstream firms can acquire. Downstream firms thus must take into account this constraint in their profit maximization:

$$\underset{H_d}{max}\ \pi_i^+|_{s.t.\ V_u \leqslant \overline{V_u}}$$

$$H_d^*(\overline{V_u}) = \frac{(2m\overline{V_u} + n^2m - nm + n^2 - n)(n - m\overline{V_u})}{n^3(m+1)} \tag{16}$$

This is the level of PHO that a retailer would choose from profit maximization if PVO were limited to $V_u = \overline{V_u} > 0$, in that case there is no social improvement in comparison to an only PHO situation. If now authorities, want to act on the welfare in an improvement purpose, they can decide to suddenly decrease the threshold of PVO (ex-post PO choices of the firms) .In this manner the amount of PVO at optimum would be lower than what retailers expected, and in consequence the level of PHO chosen by dealers would not be as large as the profit maximization's one. But by doing this, consumer surplus, as total welfare are increased.

Now imagine that the same authorities could limit the maximum value of downstream PHO when manufacturers are choosing their amount of PVO to invest in retailers:

$$\underset{V_u}{max}\ \pi_k^+|_{s.t.\ H_d \leqslant \overline{H_d}}$$

In this situation it is possible to find a $\overline{H_d}$ threshold[22] lying in the interval J for which optimal choice of $V_u^*(H_d < \overline{H_d})$ leads to better social situation.

[22] As an example, *when* $m = 3$ *and* $n = 2$, $\overline{H_d} \approxeq 31.25\%$; under this threshold we obtain a social improvement of welfare. Other numerical applications are available on request.

Authorities are no longer obliged to cheat[23] (ex-post maximization choices) in the goal of enhancing surplus and welfare. Off course above this threshold the impact on the industry is reverted with an increase of firms profits but a fall in consumer and overall welfare.

In conclusion the regulation on PHO seems to be the best tool to force firms to themselves choose a level of optimal PVO leading to a reduced collusion and to lessened losses of production.

4.3 Main indicators of Performance and Welfare

We present in the following a table grouping the main quantities of our model in which we have replaced the PO by their value at the optimum. In a comparison goal we add optimum values coming from the resolution of two other games, which are quasi similar with the one we studied all over this paper, only the financial arrangements part is modified: in the first one only downstream PHO are allowed, and the other one is a standard successive Cournot oligopoly without PO at all. The former show the impact of a mixed PHO-PVO optimum compared to a only PHO optimum, the latter serves as a benchmark.

Table II. Cournot values at optimum

	PVO+PHO	Only downstream PHO	Benchmark
Q	$\frac{m}{2(1+m)}$	$\frac{m}{2(1+m)}$	$\frac{nm}{(1+n)(1+m)}$
p_F	$\frac{m+2}{2(1+m)}$	$\frac{m+2}{2(1+m)}$	$\frac{n+m+1}{(1+n)(1+m)}$
p_I	p_I^{*}[24]	$\frac{1}{(1+m)}$	$\frac{1}{(1+m)}$
Π	$\frac{m(m+2)}{4(1+m)^2}$	$\frac{m(m+2)}{4(1+m)^2}$	$\frac{nm(n+m+1)}{(1+n)^2(1+m)^2}$
S	$\frac{m^2}{8(1+m)^2}$	$\frac{m^2}{8(1+m)^2}$	$\frac{n^2m^2}{2(1+n)^2(1+m)^2}$
W	$\frac{m(3m+4)}{8(1+m)^2}$	$\frac{m(3m+4)}{8(1+m)^2}$	$\frac{nm(nm+2n+2m+2)}{2(1+n)^2(1+m)^2}$

We pose ** for the only PHO model optimal values, and *** for the benchmark equilibrium values. For the first two columns, all values are similar excepted for the price of the input, $p_I^* < p_I^{**}$, this is the fact of PVO which, as explained in the previous section, incentivize manufacturers to decrease their operating margin in the purpose of increasing dealers' one, for finally recapture this profit from a partial rebate. Retailers' profits (not detailed here) are greater in the only PHO case when manufacturers' profits are greater in PVO+PHO case. Nevertheless the sum all firms profits in the industry remains constant in both cases. This means that we just have a translation of profits from downstream to upstream due to PVO, but no distortion. Output quantity is the same in the two first situations, this confirms the results of the comparative static about the

[23]Nevertheless reducing this ex-post level would be even more efficient from a welfare view.

[24]The expression of the intermediate price being very quite long to derive, we provide it on request.

influence of PVO on the output which truly exists and is significant. To summarize, these two first columns of the table characterize the impact of PVO in our industry: increased concentration until it reaches the monopoly level through a massive contraction of the production, and a translation of profits from retailers to producers.

We now compare the first column to the last one which contains equilibrium values of the benchmark: $Q^{***} > Q^{*}$, $S^{***} > S^{*}$. We note here the harmful influence of combined vertical and horizontal participations on market competition leading to a fall in the consumer surplus. As for horizontal shareholdings which have already raised antitrust concerns, this result should alert the authorities. Indeed an industry where there is a mixed equilibrium of vertical and horizontal stakes generates a result as hurtful to the consumer surplus as an industry with only PHO in greater quantity. This reduced competition increases whole industry profits $\Pi^{***} < \Pi^{*}$ (for $m > n > 2$) but anyway the total welfare is greater in the benchmark case, $W^{***} > W^{*}$. In other words, the increased efficiency of the industry does not compensate the loss of consumer surplus, and thus in the end the economic situation is degraded.

5 Conclusion

There are many examples of suppliers acquiring vertical stakes in their retailers, themselves linked horizontally with other retailers. It is therefore interesting to understand the incentives of firms to acquire cross shareholdings, and a good way to realize this was to develop a model.

This study highlights the existence of a strategic relation between PHO and PVO; it also provides a precise analysis of the impact of equity investments on production equilibrium in an industry composed of two Cournot successive oligopolies. We assert that PVO; as it was already proven in the literature for PHO; increase final price as a consequence of the output contraction. This similar effect of horizontal and vertical participations on output leads us to a better understanding of the strategic substitutability. This substitutability can be explained by the fact that PHO could replace PVO to achieve a particular level of collusion and vice versa. Moreover we show that a mixed equilibrium of PHO and PVO is as socially harmful as an equilibrium with only PHO.

Competition authorities are already aware of the negative effects caused by PHO, but our results should alert them also about the dangerousness of PVO. A way to deal with this problem could be to apply a regulation to the acquisition of PHO rather than to PVO, but this would penalize retailers and would profit to producers.

Appendix

Concavity of downstream firms profits subject to quantities

To prove the concavity of profits functions we examine the second order conditions then we find circumstances under which they are negative, we start with a retailer profit second derivative subject to quantities:

$$\frac{\partial^2 \pi_i}{\partial {q_i}^2} = -2 + 2(\sum_{j \neq i} h_{ji} + \sum_{k=1}^{m} v_{ki}) < 0$$

This expression is negative if $\sum_{j \neq i} h_{ji} + \sum_{k=1}^{m} v_{ki} < 1$, i.e. if the total value of shares sold by a firm is less than 100% of its capital, this condition is verified at optimum.

Concavity of equilibrium profits subject to participations

To prove the concavity of profits functions we examine the second order conditions then we find circumstances under which they are negative, we start with a dealer profit second derivative subject to PHO:

$$\frac{\partial^2 \pi_i^+}{\partial {H_d}^2} = - \frac{2n^5 m^2 (mV_u - n)^2 (1+m) \left(2n^2 - 6nmV_u - n^3 + n^2 mV_u + n^3 H_d + 2n^2 m - 2nm^2 V_u + n^2 m^2 V_u + 4m^2 {V_u}^2 + n^3 m H_d - n^3 m\right)}{(-n^2 + 3nmV_u - n^3 + n^2 mV_u + n^3 H_d - n^2 m + nm^2 V_u + n^2 m^2 V_u - 2m^2 {V_u}^2 + n^3 m H_d - n^3 m)^4}$$

We should preferentially study the sign of the last term of the numerator, all others terms being obviously positive, it has to be positive.
$\frac{\partial^2 \pi_i^+}{\partial {H_d}^2} < 0$, if $H_d > -\frac{2n^2 - 6nmV_u - n^3 + n^2 mV_u + 4m^2 V_u^2 + 2n^2 m - 2nm^2 V_u + n^2 m^2 V_u - n^3 m}{n^3(1+m)}$,
this condition is always true for $V_u \in I$, for $m \geqslant 2$ and $n \geqslant 2$.

The second derivative of a producer profit subject to PHO is also negative. This expression is quite complex to solve, but around the optimum we can affirm that $\frac{\partial^2 \pi_k^+}{\partial {V_u}^2}|_{H_d = H_d^*} < 0$.

Explicit value of PVO

This expression gives us the optimal value of PVO chosen by upstream firms as a resul of profit maximization:

$$V_u^* = \frac{-n(-m^3 + 2nm - 5m - 10 + 2n + \sqrt{4 - 8n + 4m + 4n^2 + 8n^2 m - 7m^2 + 4nm + 4n^2 m^2 - 4m^3 n - 12m^3 + 10m^4 - 4nm^4 + m^6 + 12nm^2})}{16m}$$

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